CS-211
Spring 04
Week 1-2 Solutions

(Sec. 1.3 p21)

4. A) $\{3\}$ B) $\{-3, 3\}$ c) $\{-3, 3\}$ d) $\{4, 5, 6\}$
   e) $\{-6, -5, -4, 4, 5, 6\}$ f) $\emptyset$

6. A) $\{1, 2, 3, 4, 6, 12\}$ B) $\emptyset$ C) $\{24, 25, 26\}$
   D) $\{15, 16\}$

8. A) 0 B) 74 C) 138 D) 67 E) 73 F) ∞
   G) 0 H) 2

(Sec 1.4 p27)

2. A) $A \cap B = \{2\}$, $B \cap C = \emptyset$, $B \cup C = \mathbb{P}$, $B \oplus C = \mathbb{P}$
   B) $\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$
   d) $A \oplus B$, $A \oplus C$, $C \setminus A$.

4. A) $\{a, b, aa, bb\}$, $\{aaa, bbb\}$, $\{\lambda, ab, ba\}$,
   $\{\lambda, ab, ba, aaa, bbb\}$
   B) $\{aa, bb, aaa, bbb\}$, $\{aa, ab, ba, bb\}$, Σ^*, the set of words of length 2 or more, except for aa, bb, aaa, and bbb
   c) $\{\lambda, a, b\}$, $\{a, b\}$, $\emptyset$ d) $\emptyset$, $\{a\}$, $\{b\}$, $\{a, b\}$
   e) 4

6. A) Let $A = \{1\}$, $B = \{2\}$, $A \cup B = \{1, 2\} \subseteq \emptyset = A \cap B$.
   B) Let $A = \{1\}$ $A \cap \emptyset = \emptyset \neq \{1\}$.
   c) Let $A = \emptyset$ $B = \{1\}$, $C = \{2\}$, then
   $A \cap (B \cup C) = \emptyset \neq \{2\} = \emptyset \cup \{2\} = (A \cap B) \cup C$.

8. A) 6 B) 5 C) 7 D) $6 + 5 - 4 = 7$.
   E) $|A| + |B|$ counts members of A and B, but elements in common $(A \cap B)$ got counted twice.

12. A) 15, 15.

B) (0, 2), (0,4), (1,2), (1,4), (2,4), (3,4)

C) (0,1), (0,2), (0,3), (0,4), (2,3), (2,4)

(Sec. 2.1, p. 57)

6. A) $r \to q$. b) If I am rich, then I am short

c) If $x=0$ or $x=1$, then $x^2 = x$.

D) If $2+4=8$, then $2+2=4$.

12. A) $n=6$, $2^6 - 1 = 63 = 9 \cdot 7$.

B) $n=3$, $2^3 + 3^3 = 8 + 27 = 35$

C) $n=7$ $2^7 + 7 = 128 + 7 = 135$

14. A.) TRUE. Commutative law for sets

B.) False. Let $B = \{1\}$, $A = \emptyset$.

C.) False Let $A = B$, $A \neq \emptyset$.

D.) TRUE. Associative law for sets

(Sec. 2.2 p. 65-66)

8.

p	q	$(p \to q)$	$\to$	$[(p \vee$	$\neg q)$	$\to$	$(p \vee q)]$
0	0	1	0	1	1	0	0
0	1	1	1	0	0	1	1
1	0	0	1	1	1	1	1
1	1	1	1	1	0	1	1
step		1	4	2	1	3	1

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p	q	r	$[(p \leftrightarrow q)$	$\vee$	$(p \to r)]$	$\to$	$(\neg q$	$\wedge p)$
0	0	0	1	1	1	0	1	0
0	0	1	1	1	1	0	1	0
0	1	0	0	1	1	0	0	0
0	1	1	0	1	1	0	0	0
1	0	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	1	0	0	0	0
1	1	1	1	1	1	0	0	0
	step		1	2	1	3	1	2

14 A)

p	q	$(p\wedge q)$	$\vee$	$(p\wedge$	$\neg q)$	$\leftrightarrow$	p
0	0	0	0	0	1	1	0
0	1	0	0	0	0	1	0
1	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1
		1	3	2	1	4	1

Notice that the left hand side is true only when p is true

16. A) $(p\wedge\neg q\wedge\neg r)\vee(\neg p\wedge q\wedge\neg r)\vee(\neg p\wedge\neg q\wedge r)$

B) $(p\wedge q\wedge\neg r)\vee(p\wedge\neg q\wedge r)\vee(\neg p\wedge q\wedge r)$

(Section 2.5, p 85)

10 A) Rule 22, with $\neg p\vee q$ replacing q

B) Rule 4a, with $r\wedge s$ replacing r

C) Rule 20, with $p\to s$ replacing p and $q\wedge s$ replacing q

14.

p	q	$p\to$	$[q\to$	$(p\wedge q)]$	$(p\vee q)$	$\to$	$[q\to$	$(p\vee q)$	$\wedge q]$
0	0	1	1	0	0	1	1	0	0
0	1	1	0	0	1	1	1	1	1
1	0	1	0	0	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1
		3	2	1	1		3	1	2

But, if you just replace the <u>first</u> p with $p\vee q$, then consider the following line of a truth table:

p	q	p	$\to$	$[q\to$	$(p\wedge q)]$	$(p\vee q)$	$\to$	$[q\to$	$(p\wedge q)]$
0	1	0	1	0	0	1	0	0	0

So one is a tautology, but the other isn't!

16. Work in class

18. work in class