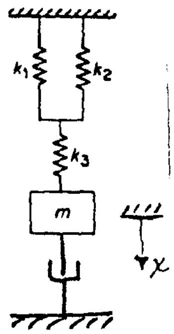
HW. 3.

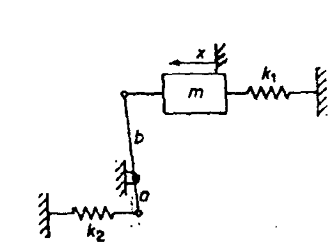
Determine the e.o.m. using energy in terms of the variable depicted (x or θ). Determine ~~variab~~ values.

①



determine ω_n, c_c, ζ, ω_d

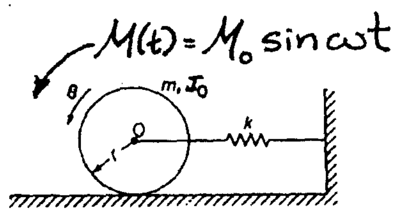
②



*neglect the mass of the lever bar.

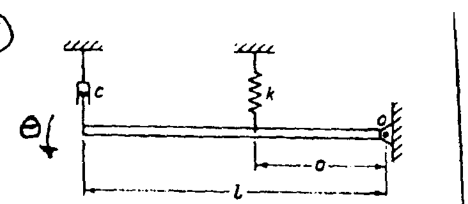
determine ω_n

③



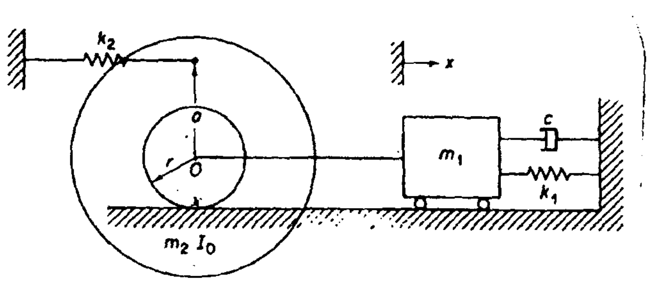
determine ω_n, r, ζ, ω_d

④



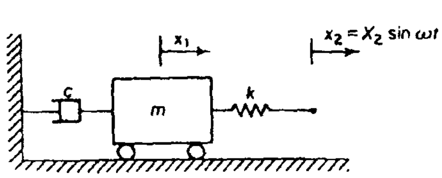
determine ω_n, c_c, ζ, ω_d

⑤



determine ω_n, ζ, ω_d

⑥



(given displacement)

determine ω_n, F_o, r, ζ, ω_d

1) $m\ddot{x} + c\dot{x} + k_{eq}x = \varnothing$

$$\frac{1}{k_{eq}} = \frac{1}{(k_1+k_2)} + \frac{1}{k_3}$$

$$= \frac{k_3}{k_3(k_1+k_2)} + \frac{(k_1+k_2)}{k_3(k_1+k_2)}$$

$$k_{eq} = \frac{k_3(k_1+k_2)}{k_1+k_2+k_3}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \left[\frac{k_3(k_1+k_2)}{m(k_1+k_2+k_3)}\right]^{1/2}$$

$$C_c = \sqrt{4mk_{eq}} = 2\sqrt{mk_{eq}} \qquad \zeta = \frac{c}{2\sqrt{mk_{eq}}} \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

2) $$\left[\frac{\left(k_1 + \frac{a^2}{b^2}k_2\right)}{m}\right]^{1/2} = \omega_n$$

3) $(m_1r^2 + I)\ddot{\theta} + kr^2\theta = M_0 \sin\omega t$

$$\omega_n = \sqrt{\frac{kr^2}{m_1r^2+I}} \qquad r = \frac{\omega}{\sqrt{\frac{kr^3}{mr^2+I}}}$$

$C = \varnothing \therefore$

$\zeta = \varnothing$ $\quad \cancel{\omega_d = \omega_n\sqrt{1-\zeta^2}}$

$$\omega_d = \omega_n$$

4) $\left(\frac{ml^2}{4} + I_{cg}\right)\ddot{\theta} + cl^2\dot{\theta} + ka^2\theta = \varnothing$

$$\omega_n = \left[\frac{ka^2}{\frac{ml^2}{4} + I_{cg}}\right]^{1/2}$$

$$(C_cl^2)^2 - 4\left(\frac{ml^2}{4} + I_{cg}\right)ka^2 = \varnothing$$

$$C_c\cancel{l^2} = \frac{2a\sqrt{\left(\frac{ml^2}{4} + I_{cg}\right)k}}{l^2}$$

$$\zeta = \frac{c}{C_c} = \frac{cl^2}{2a\sqrt{\left(\frac{ml^2}{4} + I_{cg}\right)k}}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

where ω_n & ζ are defined above

5. $(m_1+m_2+I_0/r^2)\ddot{x}+c\dot{x}+\left[k_1+k_2\left(\frac{a+r}{r}\right)^2\right]x=\emptyset$

$$\omega_n=\left[\frac{k_1+k_2\left(\frac{a+r}{r}\right)^2}{(m_1+m_2+I_0/r^2)}\right]^{1/2}$$

$$2\zeta\omega_n=\frac{c}{(m_1+m_2+I_0/r^2)}$$

$$\zeta=\frac{c}{2\left[\frac{k_1+k_2\left(\frac{a+r}{r}\right)^2}{\cancel{(m_1+m_2+I_0/r^2)}}\right]^{1/2}\left[m_1+m_2+I_0/r^2\right]^{1/2}}$$

$\omega_d=\omega_n\sqrt{1-\zeta^2}$ where ω_n & ζ are defined above

6. $\Delta T=\frac{1}{2}m\dot{x}_1^2$

$\Delta P=\frac{1}{2}c\dot{x}_1^2 \quad \therefore \Delta W=\emptyset$

$\Delta V_e=\frac{1}{2}k(x_1-x_2)^2$

$\Delta V_g=\emptyset$

$$\frac{d}{dt}\left[\frac{\partial T}{\partial\dot{x}_1}\right]-\frac{\partial T}{\partial x_1}+\frac{\partial U}{\partial x_1}+\frac{\partial P}{\partial\dot{x}_1}=Q_1=\emptyset$$

$$m\ddot{x}_1+k(x_1-x_2)+c\dot{x}_1=\emptyset$$

$$m\ddot{x}_1+c\dot{x}_1+kx_1=kx_2=k\bar{X}_2\sin\omega t$$

$\omega_n=\sqrt{\frac{k}{m}} \quad F_0=k\bar{X}_2 \quad r=\omega\left(\frac{m}{k}\right)^{1/2}=\frac{\omega}{\omega_n}$

$2\zeta\omega_n=\frac{c}{m} \quad \zeta=\frac{c}{2m^{1/2}\left(\frac{k}{\cancel{m}}\right)^{1/2}}=\frac{c}{2\sqrt{km}}$

$$\omega_d=\omega_n\sqrt{1-\zeta^2}=\left(\frac{k}{m}\right)^{1/2}\left(1-\frac{c^2}{4km}\right)^{1/2}$$