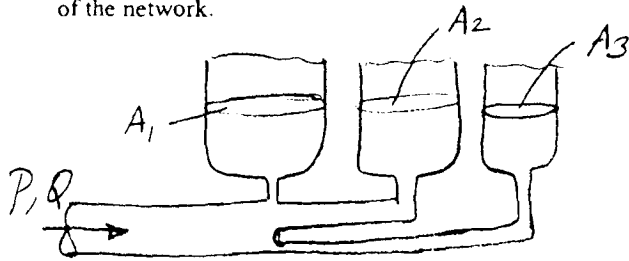
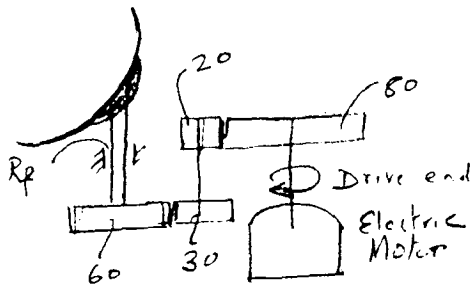


1. The schematic arrangement of three water tanks connected to a city water network is shown below. Find the equivalent compliance of the network looking from the supply end. The relevant dimensions are given below. Neglect parasitic effects such as resistance and inductance effects of the pipe. Also draw a bond graph of the network.

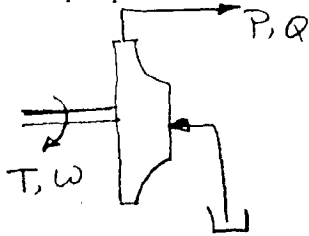


$A_1 = 4\text{m}^2, A_2 = 0.5\text{m}^2, A_3 = 0.2\text{m}^2$
 $\rho_{\text{water}} = 1\text{kg/liter}, g = 9.81\text{m/s}^2$

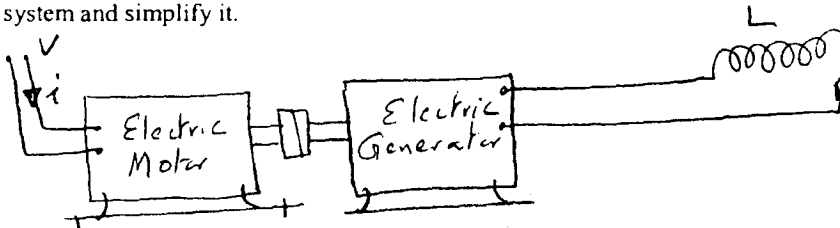
2. The arrangement of a satellite drive system is shown below. The moment of inertia of the dish and its moving parts is 4kgm^2 . The dish drive is operated by an electric motor. Find the inertia seen at the drive end. The friction factor R_f measured at the drive is observed to be 20N-m-s/rad . What is the value of the friction factor, R_f , at the load (dish) end.



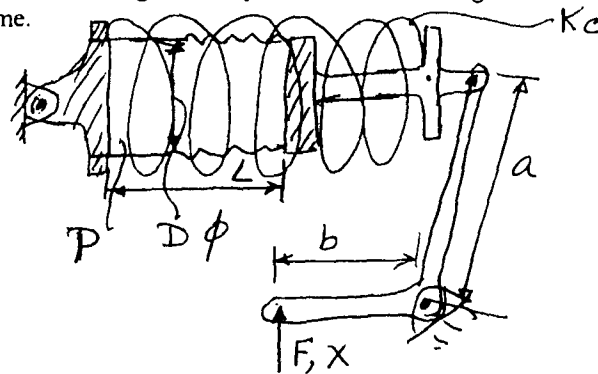
3. The schematic arrangement of an impeller pump is shown below. Draw a bondgraph model of the pump. Find the pump modulus, the pump produces 3Mpa at 1000rpm . Find the input power at the drive end when the pump delivers 50l/min .



4. Find the transformer modulus of the arrangement shown below. What is the value of the inductance seen at the input end if the load inductance connected to the generator is 50mH . Also draw a bondgraph of the system and simplify it.

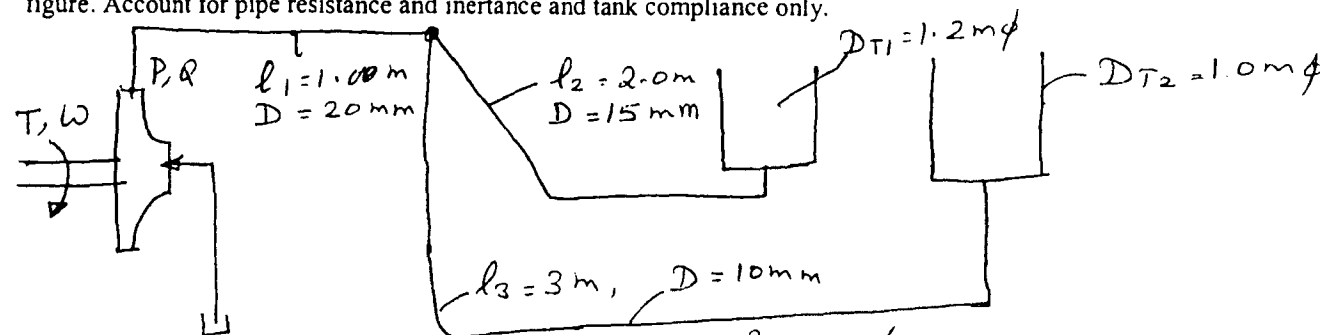


5. The arrangement of a gas spring and a coil spring combination used in a truck is shown below. Draw a bond graph of the arrangement. Find the compliance and stiffness of this arrangement at the input end indicated on the figure. The parameter values are give below. Assume adiabatic process for the gas volume.



$$\begin{aligned}
 P &= 1 \text{ MPa} \\
 D\phi &= 7 \text{ cm} \\
 L &= 15 \text{ cm} \\
 K_c &= 10,000 \text{ N/m} \\
 a &= 15 \text{ cm} \\
 b &= 7.5 \text{ cm}
 \end{aligned}$$

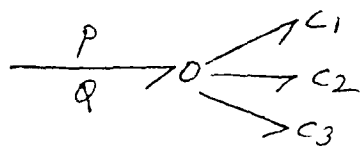
6. The arrangement of a system used for pumping water is shown below. Draw a Bond graph of the system. Also find the equivalent effects at input end (input shaft location) due the load elements shown on the figure. Account for pipe resistance and inertance and tank compliance only.



$$\begin{aligned}
 \mu &= 1.0 \times 10^{-3} \text{ N-s/m}^2 \\
 \rho &= 1000 \text{ kg/m}^3 \\
 g &= 9.81 \text{ m/s}^2
 \end{aligned}$$

ME-330P, Dynamic Systems, Home work

①

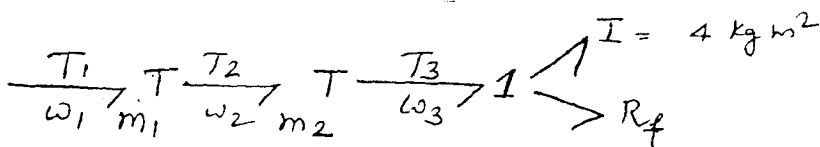


$$\equiv \frac{P}{Q} \rightarrow C_g = C_1 + C_2 + C_3$$

$$C_i = \frac{A}{\rho g}, \quad C_i = \frac{4 \times m^2}{\frac{1000 \text{ kg}}{m^3} \frac{m \times 9.81}{s^2}} = \frac{m^3 m^2 s^2}{\text{kg m}} = \frac{m^4 s}{\text{kg}}$$

$$= \frac{4}{1000 \times 9.81} + \frac{0.5}{1000 \times 9.81} + \frac{0.2}{1000 \times 9.81} = \frac{4.791 \times 10^{-4} \text{ m}^4 \text{ s}}{\text{kg}}$$

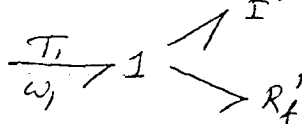
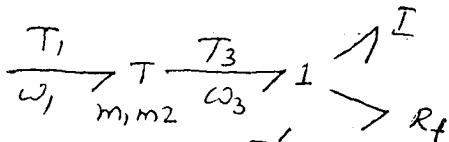
②



$$m_1 = \frac{N_1}{N_2} = \frac{80}{20} = 4$$

$$m_2 = \frac{N_3}{N_4} = \frac{30}{60} = 0.5$$

$$m = m_1 m_2 = 4 \times 0.5 = 2$$

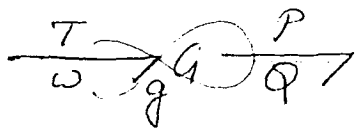


$$I' = 4 \times (2)^2 = 4 \times 4 = 16 \text{ kg m}^2$$

$$R_f' = R_f m^2, \quad R_f' = 20, \quad R_f = R_f' / m^2 = \frac{20}{2^2} = \frac{20}{4}$$

$$R_f = 5 \frac{\text{N-m-s}}{\text{rad}}$$

③



$$\omega g = P$$

$$g = \frac{P}{\omega} = \frac{3 \times 10^6 \text{ N/m}^2}{\frac{100\pi \times 2\pi \cdot \text{rad}}{60} \text{ s}}$$

$$= 2.8647 \cdot 89 \frac{\text{N s}}{\text{m}^2 \text{ rad}}$$

$$\text{Power} = P \omega = T \omega$$

$$= \frac{3 \times 10^6 \text{ N}}{\text{m}^2} \times \frac{50}{1000 \times 60} = 2500 \frac{\text{N m}}{\text{s}}$$

$$\frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^3}{\text{s}}$$

④

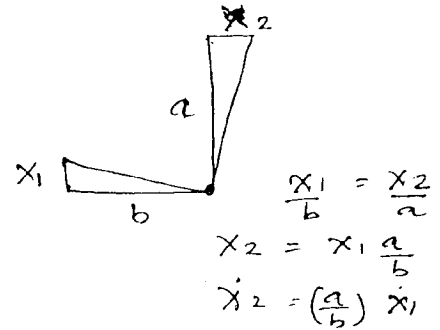
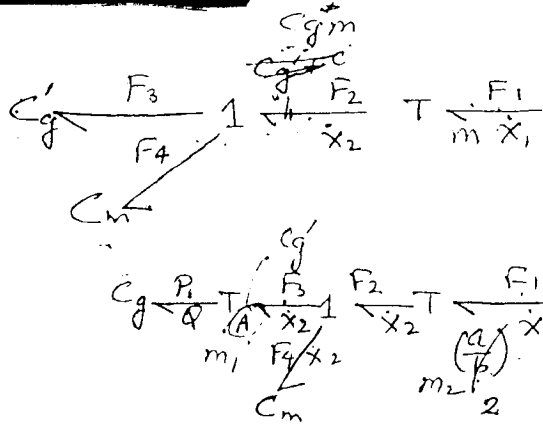
$$\frac{V}{i} \xrightarrow{g_1} \omega \xrightarrow{g_2} \frac{V}{i} \rightarrow L$$

$$\frac{V}{i} \xrightarrow{g_1/g_2} \frac{V}{i} \rightarrow L \equiv \frac{V}{i} \rightarrow L (g_1/g_2)^2$$

$$L' = L (g_1/g_2)^2 =$$

$$= 50 \text{ mH} (g_1/g_2)^2$$

5



$$\frac{x_1}{b} = \frac{x_2}{a}$$

$$x_2 = x_1 \frac{a}{b}$$

$$\dot{x}_2 = \left(\frac{a}{b}\right) \dot{x}_1$$

$$\frac{a}{b} = \frac{15}{7.5} = 2$$

$$A = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4} \left(\frac{7}{100}\right)^2$$

$$= 0.0038 \text{ m}^2$$

$$= \underline{3.8 \times 10^{-3} \text{ m}^2}$$

$$C_g = \frac{V}{P} = \frac{V}{rP}$$

$$r = 1.4$$

$$P = 1 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

$$C_g' = \frac{C_g}{m^2} = \frac{V}{rPA^2} = \frac{rL}{rPA^2} = \frac{L}{rPA}$$

$$C_{gm}' = \left(\frac{1}{\frac{1}{C_g'} + \frac{1}{C_{gm}}} \right) =$$

$$C_{gm}' = \frac{C_{gm}}{m_2^2} = \frac{C_{gm}}{2^2} = \frac{1}{4} (C_{gm})$$

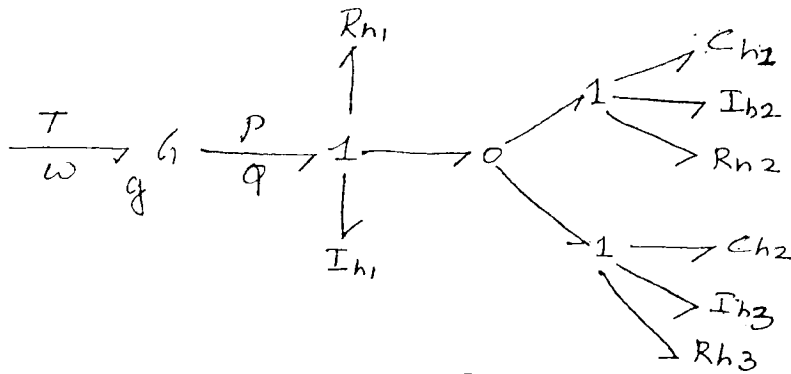
$$C_g' = \frac{L}{rPA} = \frac{0.15 \text{ m}}{1.4 \times \frac{1 \times 10^6 \text{ N}}{\text{m}^2} \times 3.8 \times 10^{-3} \text{ m}^2} = \underline{2.8 \times 10^{-5} \text{ m/N}}$$

$$C_{gm} = \frac{1}{\frac{1}{2.8 \times 10^{-5}} + 10,000} = \underline{2.19 \times 10^{-5} \text{ m/N}}$$

$$C_{gm}' = \frac{2.19 \times 10^{-5} \text{ m/N}}{4} = \underline{0.549 \times 10^{-5} \text{ m/N}}$$

$$K_{gm} = \frac{1}{C_{gm}'} \approx \underline{\underline{182,000 \text{ N/m}}}$$

6



$$Ch_1 = \frac{A}{\rho g} = \frac{\frac{\pi}{4} \times 1.2^2}{1000 \times 9.81} = \frac{m^2}{kg/m^3 \cdot \frac{m}{s^2}} = \frac{m^2 m^3 s^2}{kg} = \frac{m^5 s^2}{kg}$$

$$= 1.153 \times 10^{-4} \text{ m}^5 \text{ s}^2 / \text{kg}$$

$$Ch_2 = \frac{\frac{\pi}{4} \times 1^2}{1000 \times 9.81} = 0.800 \times 10^{-4} \frac{\text{m}^5 \text{s}^2}{\text{kg}}$$

$$I_{h1} = \frac{\rho l}{A} = \frac{1000 \times 1}{\frac{\pi}{4} \times \left(\frac{20}{1000}\right)^2} = \frac{kg}{m^3} \frac{m}{m^2} = kg/m^4$$

$$= 3183098 \text{ kg/m}^4$$

$$I_{h2} = \frac{\rho l}{A} = \frac{1000 \times 2}{\frac{\pi}{4} \times \left(\frac{15}{1000}\right)^2} = 11317684 \text{ kg/m}^4$$

$$I_{h3} = \frac{\rho l}{A} = \frac{1000 \times 3}{\frac{\pi}{4} \times \left(\frac{10}{1000}\right)^2} = 38197186 \text{ kg/m}^4$$

$$R_{h1} = \frac{8 \mu l}{\pi R^4} = \frac{8 \mu l}{\pi (D/2)^4} = \frac{128 \mu l}{\pi D^4} \frac{N \cdot s \cdot m}{m^2 \cdot m^4} = N \cdot s / m^5$$

$$R_{h1} = \frac{128 \times 1 \times 10^{-3} \times 1.0}{\pi \times (20/1000)^4} = 25464.7 \text{ N} \cdot \text{s} / \text{m}^5$$

$$R_{h2} = \frac{128 \times 10^{-3} \times 2}{\pi \times (15/1000)^4} = \frac{1609626}{121575} \text{ N} \cdot \text{s} / \text{m}^5$$

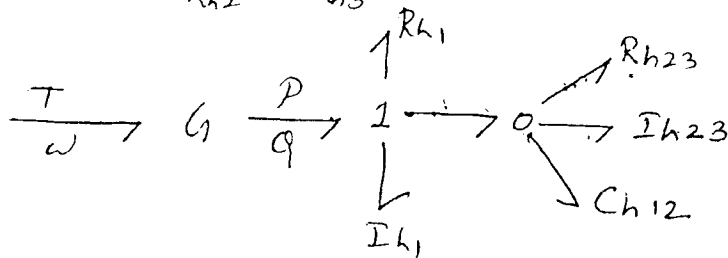
$$R_{h3} = \frac{128 \times 10^{-3} \times 3}{\pi \times (10/1000)^4} = 12223099 \text{ N} \cdot \text{s} / \text{m}^5$$

$$Ch_{12} = Ch_1 + Ch_2 (\text{at junction}) = (1.153 + 0.8) \times 10^{-4} \frac{\text{m}^5 \text{s}^2}{\text{kg}}$$

$$= 1.953 \times 10^{-4} \frac{\text{m}^5 \text{s}^2}{\text{kg}}$$

$$I_{h23} = \frac{1}{\frac{1}{I_{h2}} + \frac{1}{I_{h3}}} (\text{at junction}) = 8730784 \frac{\text{kg}}{\text{m}^4}$$

$$R_{h23} = \frac{1}{\frac{1}{R_{h2}} + \frac{1}{R_{h3}}} = 1422324.0 \frac{N \cdot s}{m^5}$$

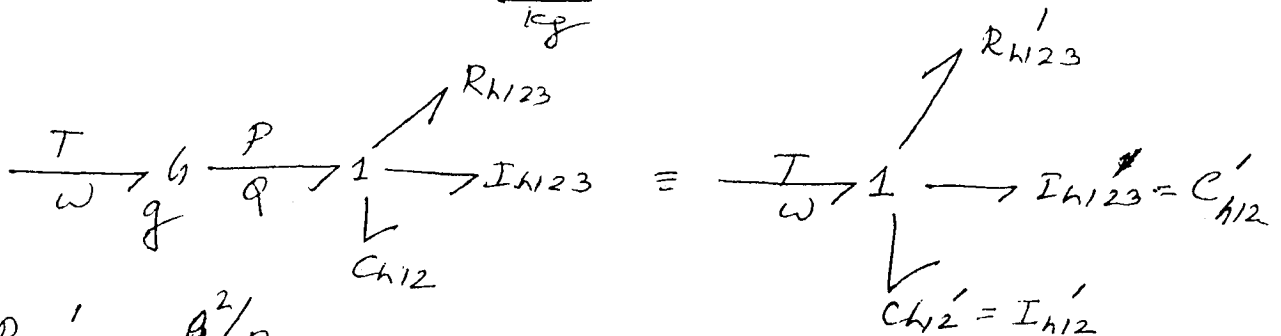


$$R_{h123} = R_{h1} + R_{h23} = 254647 + 1422324 = 1676971 \frac{N \cdot s}{m^5}$$

(T-junction)

$$I_{h23} = I_{h1} + I_{h23} = 3183098 + 8730784 = 11913882 \frac{kg}{m^4}$$

$$C_{h12} = 1.953 \times 10^{-4} \frac{m^4 \cdot s^2}{kg}$$



$$R_{h'123} = g^2 / R_{h123}$$

$$= g^2 / 1676971$$

$$C_{h'12} = I_{h'12} = g^2 C = g^2 (1.953 \times 10^{-4})$$

$$C_{h'12} = I / g^2 = \frac{11913882}{g^2}$$