1.5

Danial Sullivan

Machine Design MECH-312 Winter 2003 Dr. Jacqueline El-Sayed

A window manufacturer requires a crank to be designed for their casement style windows. The window crank must meet deflection, static and fatigue loading criteria. It should be designed to withstand a horizontal 1 kN force applied to the handle of the crank as shown. Both shafts have diameters of 30 mm. (1010 hot rolled Carbon Steel is used. Use the material properties from Appendix C.)

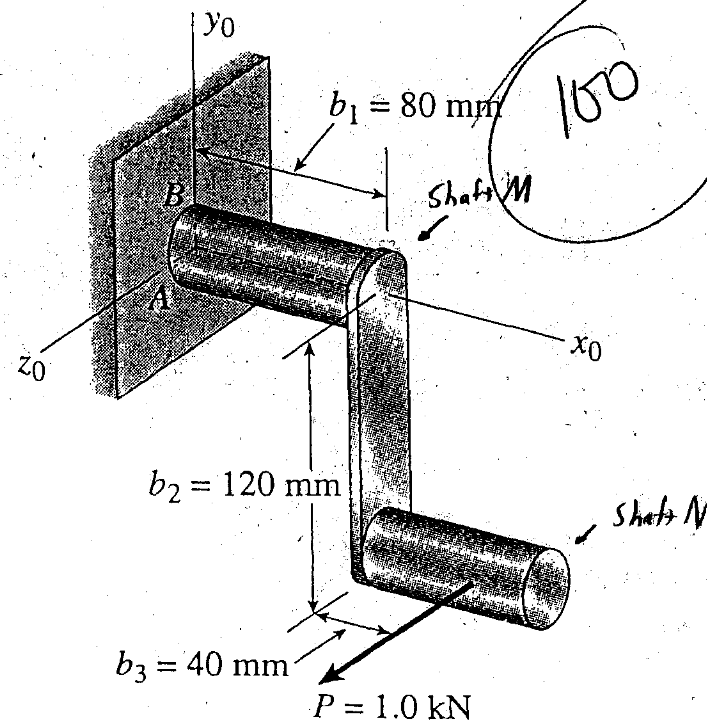
$S_{ut} = 324$ MPa $\theta = \frac{Tl}{JG}$

$G = 80.8$ GPa

Yield strength $S_y = 179$ MPa

10

100



a) Find the torsional deflection of the shafts.

b) Determine the principal stresses and the maximum shear stress at point A. Show these on Mohr's circle.

c) Determine the Von Mises stress at point A. What does the Von Mises stress represent?

d) Calculate the safety factor using the maximum shear-stress and distortion energy theories.

e) Due to the opening and closing of the window, the bending stress is fully reversed and changes from σ to $-\sigma$. Assuming that the Von Mises stress takes the sign of the fluctuating bending stress, determine the factor of safety. (The operating temperature is 120°C maximum, and the surface is machined.)

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a) $\theta = \frac{T\,l}{J\,G}$ $T_{shaft\,M} = P\,b_2$ $J_m = \frac{\pi (d_m)^4}{32}$

$l_m = b_1$ $G_m = 80.8\ GPa$ 1010 HR

$$\theta = \frac{P b_2\, b_1 (32)}{\pi d_m^4 (80.8\times10^9)} = \frac{(1.0\times10^3 N)(.08m)(.12m)(32)}{\pi(.03m)^4(80.8\times10^9)\frac{N}{m}} = 1.5\times10^{-3}\ rad$$

Ok

Not $T_{shaft\,N}$ ...

b) For A:

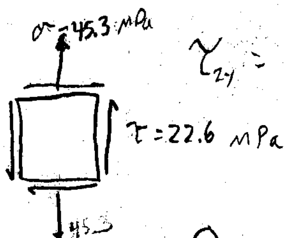
$$\sigma_x = \frac{My}{I} = \frac{(32)(1.0kN)(.04m + .08m)}{\pi(.03)^3}$$ $y=\frac{d}{2}$ $I=\frac{\pi d^4}{64}$ $\frac{y}{I}=\frac{32}{\pi d^3}$

$$\sigma_x = 45.3\ MPa$$

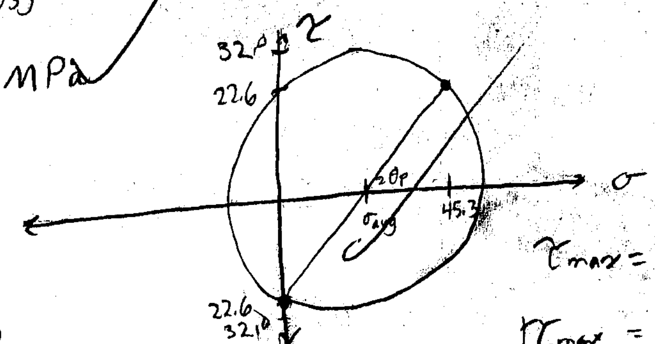
$$\tau_{zy} = \frac{Tr}{J} = \frac{(16)(1.0kN)(.12m)}{\pi(.03)^3}$$ $r=\frac{d}{2}$ $J=\frac{\pi d^4}{32}$ $\frac{r}{J}=\frac{16}{\pi d^3}$

$$\tau_{zy} = 22.6\ MPa$$

Ok



$\sigma_y = 0$ since $M = 0$ in the y-direction



$$\sigma_{avg} = \frac{45.3 + 0}{2}$$

$$\sigma_{avg} = 22.6\ MPa$$

$$\tau_{max} = R = \sqrt{(\sigma_x - \sigma_{avg})^2 + \tau_{zy}^2}$$

$$\boxed{\tau_{max} = R = 32.0\ MPa}$$

$$\sigma_{max} = R + \sigma_{avg} = 22.6 + 32.0$$

$$\boxed{\sigma_{max} = 54.6\ MPa}$$

$$\sigma_{min} = \sigma_{avg} - R = 22.6 - 32.0$$

$$\boxed{\sigma_{min} = -9.4\ MPa}$$

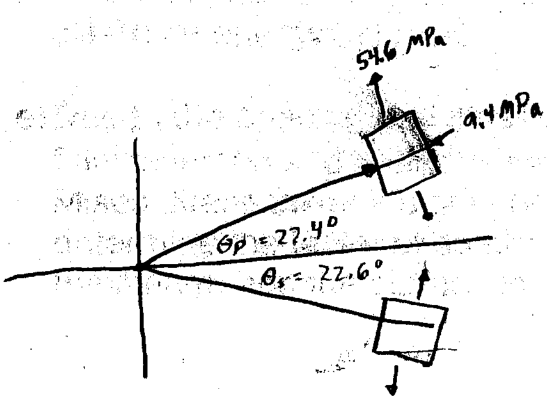
$$\tan 2\theta_p = \frac{\tau_{zy}}{\sigma_x - \sigma_{avg}}$$

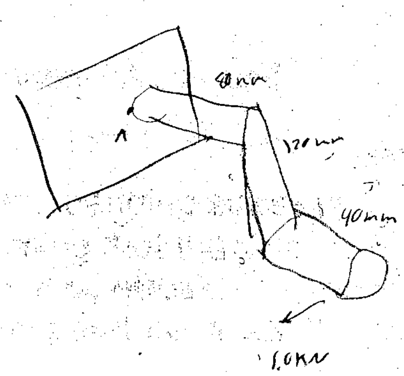
$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{22.6}{22.7}\right)$$

$$\theta_p = 22.4^\circ$$

$$\theta_s = 45 - \theta_p$$

$$\theta_s = 22.6^\circ$$





c) Von Mises

$$\sigma'_{xy} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

$$\sigma'_{xy} = \sqrt{(45.3\,MPa)^2 + 3(22.1\,MPa)^2}$$

$$\sigma'_{xy} = 59.9\ MPa$$

The Von Mises effective stress σ' is the uniaxial tensile stress that would create the same distortion energy as the actual combination of applied stresses.

d) Maximum Shear Stress

$$N = \frac{S_{ys}}{\tau_{max}} = \frac{S_y}{(\sigma_1 - \sigma_3)} = \frac{179\ MPa}{(54.6 - (-9.4))\ MPa}$$

$$\boxed{N = 2.75}$$

Distortion - Energy Theory

$$N = \frac{\overset{\text{Yield strength}}{S_y}}{\sigma'} = \frac{179\ MPa}{(59.9\ MPa)} = \boxed{2.99 = N}$$

(calculated above) Von Mises effective stress

e) In fully reversed $\sigma'_{alt} = \sigma'$; $\sigma'_m = 0$ since $-\sigma + \sigma = 0$

Factor of Safety in Fatigue $N_f = \dfrac{S_{ut}\, S_e}{\sigma'_a S_{ut} + \sigma'_m S_e}$

Endurance Limit

$$S_e = S_e'\, C_{load}\, C_{size}\, C_{surface}\, C_{temp}\, C_{reliab}$$

$$S_e = (162)(1)(.85)(.97)(1)(1) = 133\ MPa$$

$S_e' = .5\,S_{ut}$ (steel)

$S_e' = 162\ MPa$

$C_{load} = 1$ (bending)

$C_{size} = 1.189(30)^{-.097}$ (since $d = 30\,mm$)

$C_{size} = .85$

$C_{surface} = 4.51(324\,MPa)^{-.265}$ (machined)

$C_{surf} = .97$

$C_{temp} = 1$ ($T \le 450°C$)

$C_{reliab} = 1$ assume 50% reliability

Von Mises (see above)

$\sigma'_a = 59.9\ MPa$

$$N_f = \frac{(324\ MPa)(133\ MPa)}{(59.9\ MPa)(324\ MPa) + (0\ MPa)(133\ MPa)}$$

$$\boxed{N_f = 2.22}$$

ok