

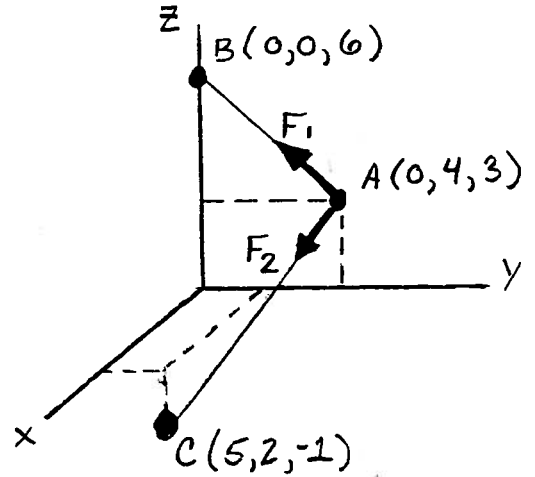
Given the three points A (0,4,3), B (0,0,6), and C (5,2,-1), in meters, and the forces  $F_1$  of magnitude 50 N in the direction of AB and  $F_2$  of magnitude 100 N in the direction of AC, as shown:

a.) Calculate the vector sum  $F_1 + F_2$

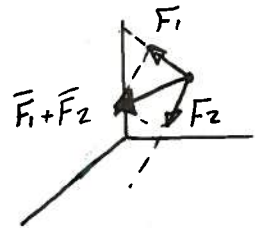
$$\vec{F}_1 = 50 \text{ N} \left( \frac{-4\hat{j} + 3\hat{k}}{5} \right) = -40\hat{j} + 30\hat{k} \text{ N}$$

$$\vec{F}_2 = 100 \text{ N} \left( \frac{(5-0)\hat{i} + (2-4)\hat{j} + (-1-3)\hat{k}}{\sqrt{5^2 + 2^2 + 4^2}} \right)$$

$$= 74.54\hat{i} - 29.81\hat{j} - 59.63\hat{k} \text{ N}$$



$$\vec{F}_1 + \vec{F}_2 = (74.54\hat{i} + (-40 - 29.81)\hat{j} + (30 - 59.63)\hat{k}) \text{ N}$$

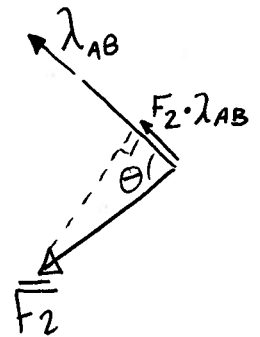


$$F_1 + F_2 = \underline{74.5} \text{ i} - \underline{69.8} \text{ j} - \underline{29.6} \text{ k N}$$

b.) Calculate the component of  $F_2$  in the AB direction.

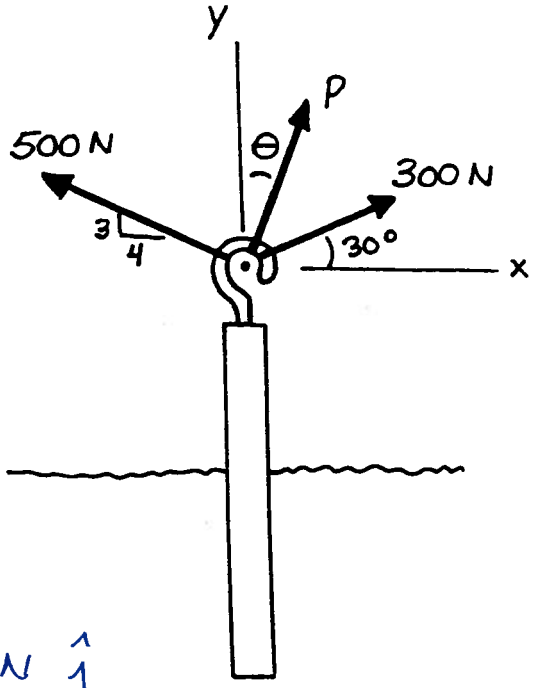
$$\vec{F}_2 \cdot \hat{\lambda}_{AB} = (74.54\hat{i} - 29.81\hat{j} - 59.63\hat{k}) \cdot (-0.8\hat{j} + 0.6\hat{k})$$

$$= (-29.81)(-0.8) + (-59.63)(0.6) = -11.93$$



$$\underline{-11.93} \text{ N}$$

Three forces are applied at the end of a hook in order to remove the post shown. Determine the magnitude  $P$  and the angle  $\theta$  of the force so that the resultant of these three forces is a 1 kN vertical force acting straight up.

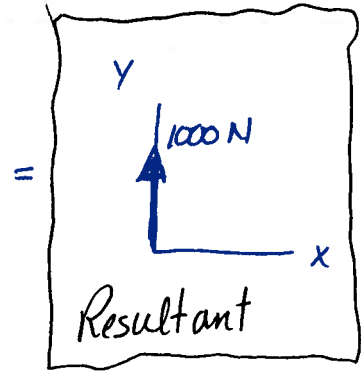
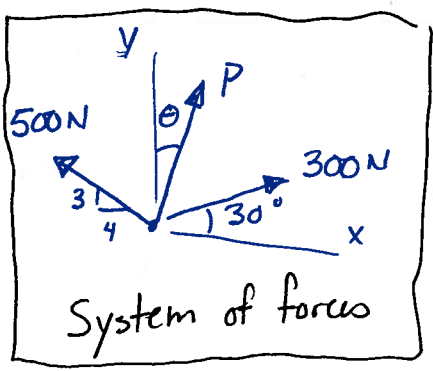


Given:  $\vec{R} = \sum \vec{F} = 1 \text{ kN } \hat{j}$  Find:  $P, \theta$

$$300 \cos 30 \hat{i} + 300 \sin 30 \hat{j} + P \sin \theta \hat{i} + P \cos \theta \hat{j} - \frac{4}{5}(500) \hat{i} + \frac{3}{5}(500) \hat{j} = 1000 \text{ N } \hat{j}$$

$$i: 300 \cos 30 + P \sin \theta - 400 = 0 \quad P \sin \theta = 140.19$$

$$j: 300 \sin 30 + P \cos \theta + 300 = 1000 \quad P \cos \theta = 550.0$$



$$\frac{P \sin \theta}{P \cos \theta} = \frac{140.19}{550.0} = \tan \theta$$

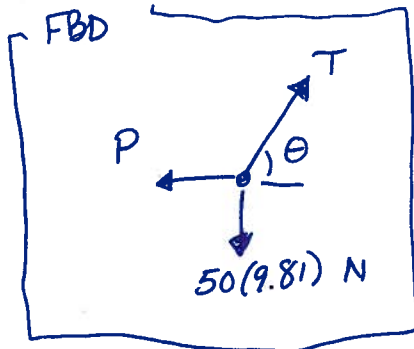
$$\theta = 14.3^\circ$$

$$P = \frac{550}{\cos 14.3} \quad P = 567.6 \text{ N}$$

$P =$	<u>568 N</u>
$\theta =$	<u>14.3°</u>

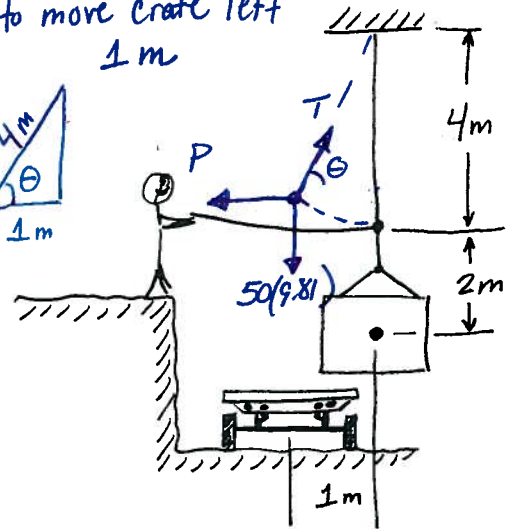
What horizontal force,  $P$ , must a worker exert on the rope to position the 50-kg crate directly over the trailer?

Given:  $M_{\text{crate}} = 50 \text{ kg}$   
 Find:  $P$  to move crate left



$$\cos \theta = \frac{1 \text{ m}}{4 \text{ m}}$$

$$\theta = 75.5^\circ$$



$$+\uparrow \Sigma F_y = 0 = \frac{T \sin 75.5}{50(9.81) \text{ N}}$$

$$T = 506.6 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0 = 506.6 \cos 75.5 - P = 0$$

$$P = 126.9 \text{ N}$$

$P = 126.9 \text{ N}$
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A cable stretched from A to B supports the 15 ft pole shown. If the tension in the cable is known to be 570 lb, calculate the moment produced by the cable tension about point C

Given:  $T_{AB} = 570 \text{ lb}$

Find:  $M_{CD}$

First find  $\bar{M}_C$ , then  $M_{CD} = \bar{M}_C \cdot \hat{\lambda}_{CD}$

$$\bar{M}_C = \bar{r}_{CA} \times \bar{T}_{AB}$$

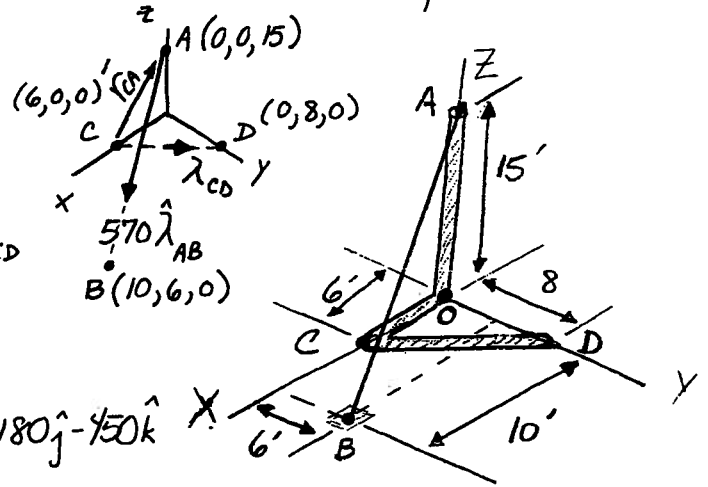
$$\bar{T}_{AB} = 570 \text{ lb} \left( \frac{10\hat{i} + 6\hat{j} - 15\hat{k}}{\sqrt{10^2 + 6^2 + 15^2}} \right) = 300\hat{i} + 180\hat{j} - 450\hat{k}$$

(check  $|T| = \sqrt{300^2 + 180^2 + 450^2} = 570 \text{ ok}$ )

$$\bar{M}_C = (-6\hat{i} + 15\hat{k}) \times (300\hat{i} + 180\hat{j} - 450\hat{k})$$

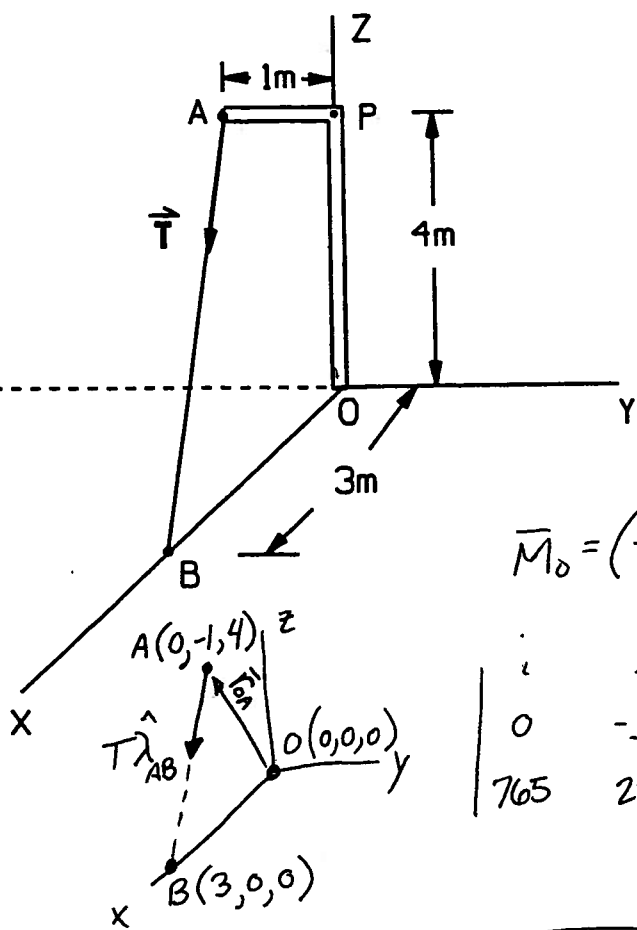
$$\begin{vmatrix} i & j & k \\ -6 & 0 & 15 \\ 300 & 180 & -450 \end{vmatrix} = [(0)(-450) - (15)(180)]\hat{i} - [(-6)(-450) - (15)(300)]\hat{j} + [(-6)(180) - (0)(300)]\hat{k} \text{ lb}\cdot\text{ft}$$

$$\bar{M}_C = -2700\hat{i} + 1800\hat{j} - 1080\hat{k} \text{ lb}\cdot\text{ft}$$



The tip of a lamp post, whose horizontal bar PA is parallel to the y-axis, is attached to point B on the x-axis by a cable AB such that a force  $T = 1.3$  kN acts along AB.

- a.) express force  $\vec{T}$  in vectorial form
- b.) find the moment vector of the force at point O
- c.) find the moments about the z-axis and about the AB-axis



Given:  $|T| = 1.3$  kN  
 Find:  $\vec{T}, M_o, M_z, M_{AB}$

$$\vec{T} = 1,300 \text{ N } \lambda_{AB}$$

$$= 1300 \left( \frac{3\hat{i} + 1\hat{j} - 4\hat{k}}{\sqrt{26}} \right)$$

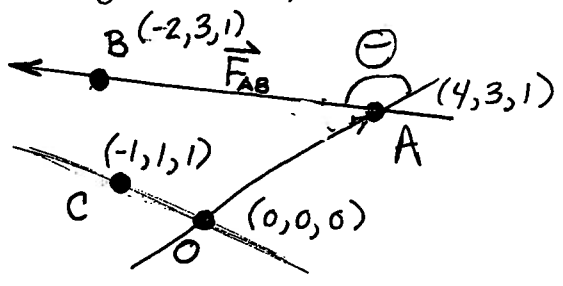
$$\vec{T} = 764.9\hat{i} + 255.0\hat{j} - 1020\hat{k} \text{ N}$$

$$\vec{M}_o = (-1\hat{j} + 4\hat{k}) \times \vec{T}$$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 4 \\ 765 & 255 & -1020 \end{vmatrix}$	$= [(-1)(-1020) - (4)(255)]\hat{i}$
	$- [(0)(-1020) - 4(765)]\hat{j}$
	$+ [(0)(255) - (-1)(765)]\hat{k} \text{ N}\cdot\text{m}$

$$M_o = 3060\hat{j} + 765\hat{k} \text{ N}\cdot\text{m}$$

Consider points A (4,3,1), B (-2,3,1), C (-1,1,1), and O (0,0,0), in meters, and the force  $\vec{F}_{AB}$  is of magnitude 80 N, as shown: Given  $F_{AB} = 80 \text{ N}$  + point coordinates



a.) Determine the angle between vector  $\vec{OA}$  and  $\vec{F}_{AB}$   
 Find  $\theta \Rightarrow$  use  $\vec{AB} \cdot \vec{AO} = |\vec{AB}| |\vec{AO}| \cos \theta$

ANSWER  
 Angle =  $38.3^\circ$

$$\vec{AB} = (-2-4)\hat{i} + (3-3)\hat{j} + (1-1)\hat{k} = -6\hat{i} \quad |\vec{AB}| = 6$$

$$\vec{OA} = (4-0)\hat{i} + (3-0)\hat{j} + (1-0)\hat{k} = 4\hat{i} + 3\hat{j} + 1\hat{k} \quad |\vec{OA}| = 5.10$$

$$\vec{AB} \cdot \vec{OA} = (-6)(4) + (0)(3) + (0)(1) = -24 \quad \cos \theta = \frac{-24}{(6)(5.10)}$$

$$\theta = 141.7^\circ$$

b.) Determine the moment of force  $\vec{F}_{AB}$  about point C

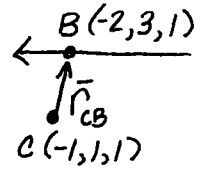
Find:  $M_C = \vec{r}_{CB} \times \vec{F}_{AB}$

ANSWER  
 $\vec{M}_C = 160 \hat{k} \text{ N}\cdot\text{m}$

$$\vec{F}_{AB} = 80 \left( \frac{-6\hat{i}}{6} \right) = -80\hat{i}$$

$$\vec{r}_{CB} = (-2-(-1))\hat{i} + (3-1)\hat{j} + (1-1)\hat{k} \text{ m}$$

$$= -1\hat{i} + 2\hat{j} \text{ m}$$

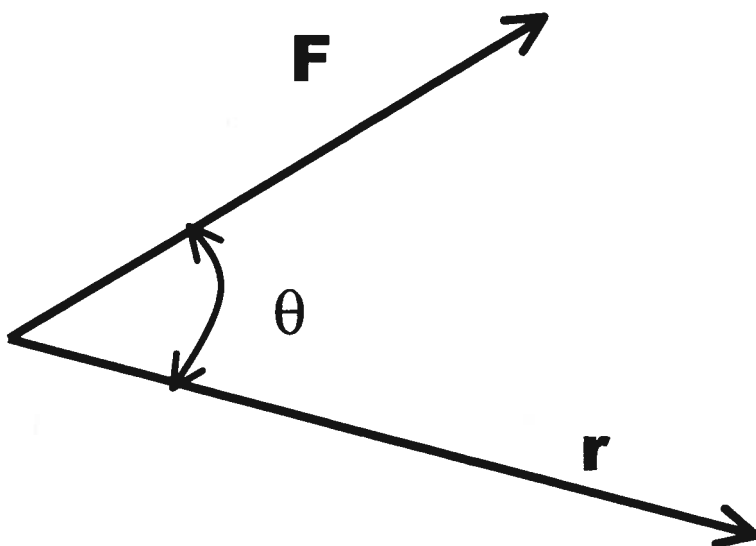


$$M_C = (-1\hat{i} + 2\hat{j}) \text{ m} \times (-80\hat{i}) \text{ N} = (2)(-80)(-\hat{k}) = 160\hat{k} \text{ N}\cdot\text{m}$$

Concept Question. Circle the correct answer.

Two vectors,  $\vec{r}$  and  $\vec{F}$ , are shown in the figure. As  $\theta$  increases from  $0^\circ$  to  $90^\circ$ ,

- a. both the magnitudes of  $\vec{r} \times \vec{F}$  and  $\vec{r} \cdot \vec{F}$  will increase but neither magnitude is greater than  $|\vec{r}| |\vec{F}|$ .
- b. both the magnitudes of  $\vec{r} \times \vec{F}$  and  $\vec{r} \cdot \vec{F}$  will decrease but neither magnitude is greater than  $|\vec{r}| |\vec{F}|$ .
- c. neither of the magnitudes of  $\vec{r} \times \vec{F}$  and  $\vec{r} \cdot \vec{F}$  change as they are independent of  $\theta$ .
- d. the magnitude of  $\vec{r} \times \vec{F}$  increases and the magnitude of  $\vec{r} \cdot \vec{F}$  decreases but neither magnitude is greater than  $|\vec{r}| |\vec{F}|$ .

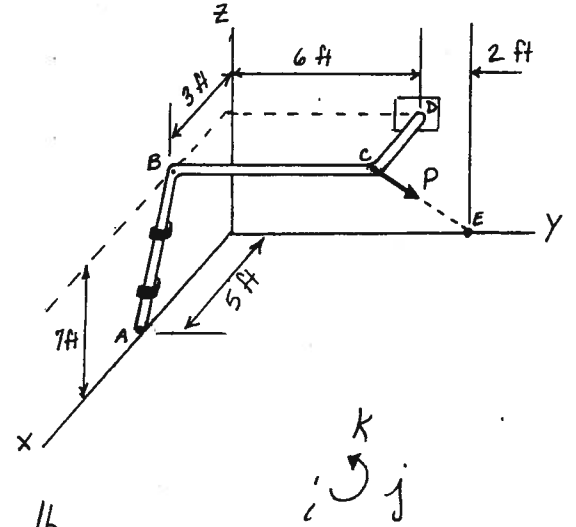


A section of bent hot water pipe is secured by brackets to a vertical wall along the section AB. A cable attached to the pipe at C exerts a force  $\mathbf{P}$  of magnitude 240 lb along the line CE as shown. Write the vector equation for the force  $\mathbf{P}$ , and calculate both the vector moment of force  $\mathbf{P}$  about point B and the magnitude of the moment of force  $\mathbf{P}$  about the axis AB.

GIVEN:  $P = 240 \text{ lb}$

FIND:  $\bar{\mathbf{P}}$ ,  $\bar{\mathbf{M}}_B$ ,  $M_{AB}$

$$\hat{\lambda}_{CE} = \frac{-3\hat{i} + 2\hat{j} - 7\hat{k}}{\sqrt{3^2 + 2^2 + 7^2}} = -0.381\hat{i} + 0.254\hat{j} - 0.889\hat{k}$$



$$\bar{\mathbf{P}} = 240 \hat{\lambda}_{CE} = -91.4\hat{i} + 61.0\hat{j} - 213.4\hat{k} \text{ lb}$$

$$\begin{aligned} \bar{\mathbf{M}}_B &= \bar{\mathbf{r}}_{BC} \times \bar{\mathbf{P}} = (6\hat{j} \text{ ft}) \times (-91.4\hat{i} + 61.0\hat{j} - 213.4\hat{k}) \text{ lb} \\ &= (6 \times -91.4)(\hat{j} \times \hat{i}) + (6)(61.0)(\hat{j} \times \hat{j}) + (6 \times -213.4)(\hat{j} \times \hat{k}) \text{ ft}\cdot\text{lb} \end{aligned}$$

$$\bar{\mathbf{M}}_B = -1280\hat{i} + 548\hat{k} \text{ ft}\cdot\text{lb}$$

$$\hat{\lambda}_{AB} = \frac{-2\hat{i} + 7\hat{k}}{\sqrt{2^2 + 7^2}} = -0.275\hat{i} + 0.962\hat{k}$$

$$M_{AB} = \bar{\mathbf{M}}_B \cdot \hat{\lambda}_{AB}$$

$$= (-1280\hat{i} + 548\hat{k}) \cdot (-0.275\hat{i} + 0.962\hat{k})$$

$$= (-1280)(-0.275) + (548)(0.962)$$

$$= +879 \text{ ft}\cdot\text{lb}$$

$$\bar{\mathbf{P}} = -91.4\hat{i} + 61.0\hat{j} - 213.4\hat{k} \text{ lb}$$

$$\bar{\mathbf{M}}_B = -1280\hat{i} + 548\hat{k} \text{ ft}\cdot\text{lb}$$

$$|M_{AB}| = 879 \text{ ft}\cdot\text{lb}$$

A chain attached to the wall at D and the hinged shelf at C has a tension of 300 lb.

(a.) Find the moment of this tension about point E

$$\vec{M}_E = \vec{r}_{EC} \times \vec{T}$$


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$$\vec{r}_{EC} = -15\vec{i} \text{ [ft]}$$

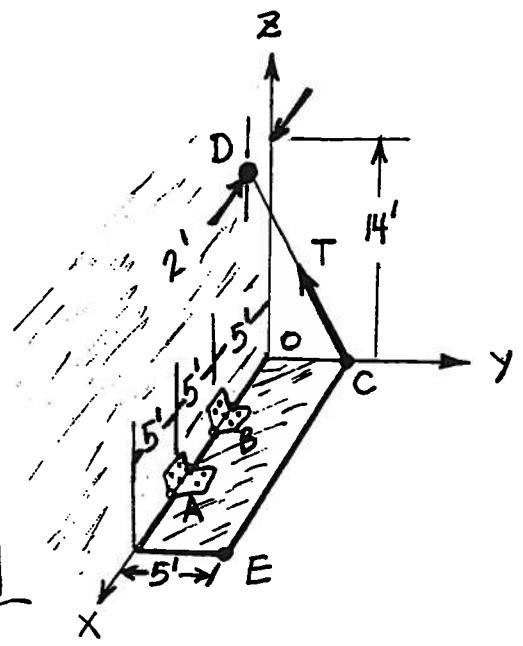
$$\vec{T} = 300 \left( \frac{2\vec{i} - 5\vec{j} + 14\vec{k}}{\sqrt{4+25+196}} \right) = \frac{300}{15} (2\vec{i} - 5\vec{j} + 14\vec{k}) \text{ [lb]}$$

$$= 40\vec{i} - 100\vec{j} + 280\vec{k}$$


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$$\vec{M}_E = 20 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -15 & 0 & 0 \\ 2 & -5 & 14 \end{vmatrix} = 20 \left[ 0\vec{i} - \cancel{(-210)}\vec{j} + 75\vec{k} \right]$$

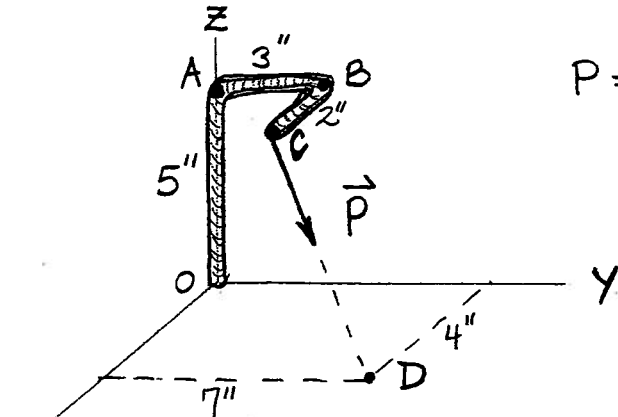
$$\vec{M}_E = 4200\vec{j} + 1500\vec{k} \text{ [lb}\cdot\text{ft]}$$



$$\vec{M}_E = \underline{4200\vec{j} + 1500\vec{k} \text{ [lb}\cdot\text{ft]}}$$

For the bracket as shown:

a.) Determine the moment of  $\vec{P}$  about the origin. FIND:  $\vec{M}_o$



$$P = 60 \text{ lb}$$

$$\vec{M}_o = \vec{r}_{OD} \times \vec{P}$$

$$\vec{r}_{OD} = 4\hat{i} + 7\hat{j} \text{ in.}$$

$$\vec{P} = 60 \left( \frac{2\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{2^2 + 4^2 + 5^2}} \right)$$

$$= 17.89\hat{i} + 35.78\hat{j} - 44.72\hat{k} \text{ lb.}$$

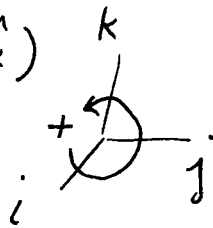
$\vec{AB}$  and  $\vec{BC}$  along principal axis directions

$$\vec{M}_o = (4\hat{i} + 7\hat{j}) \times \vec{P}$$

$$M_o = (4 \times 17.89)(\hat{j} \times \hat{i}) + (4 \times 35.78)(\hat{i} \times \hat{j}) + (4 \times -44.72)(\hat{i} \times \hat{k}) \\ + (7 \times 17.89)(\hat{j} \times \hat{i}) + (7 \times 35.78)(\hat{j} \times \hat{j}) + (7 \times -44.72)(\hat{j} \times \hat{k})$$

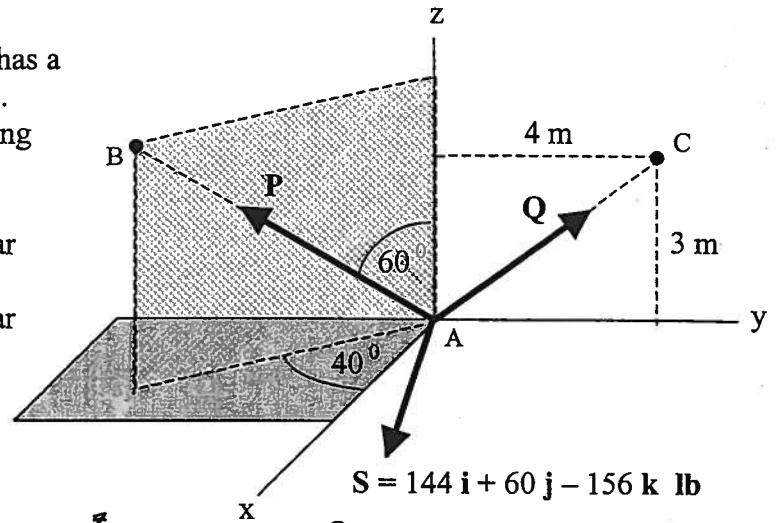
$$\vec{M}_o = -313\hat{i} + 178.9\hat{j} + 17.9\hat{k} \text{ m. lb}$$

$$\text{check } \vec{r}_{OC} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 17.89 & 35.78 & -44.72 \end{vmatrix} = -313\hat{i} + 179\hat{j} + 17.6\hat{k} \quad \checkmark$$



Three forces act at point A as shown. Force P has a magnitude of 600 lb and acts along the line AB. Force Q has a magnitude of 200 lb and acts along the line AC. And force S is described by the equation shown in the figure.

- Determine the vector equation in rectangular components for P.
- Determine the vector equation in rectangular components for Q.
- Determine the magnitude of force S.



Given:  $\vec{P} = 600 \text{ lb } \hat{\lambda}_{AB}$

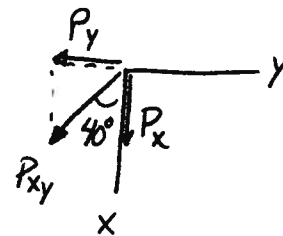
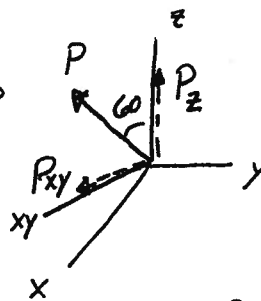
$\vec{Q} = 200 \text{ lb } \hat{\lambda}_{AC}$

$\vec{S} = 144\hat{i} + 60\hat{j} - 156\hat{k} \text{ lb}$

a) Find:  $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

$P_z = 600 \cos 60 = 300 \text{ lb}$

$P_{xy} = 600 \sin 60 = 519.6 \text{ lb}$



$P_x = P_{xy} \cos 40 = 519.6 \cos 40 = 398.0 \text{ lb}$

$P_y = 519.6 \sin 40 = 334.0 \text{ lb}$  (negative direction)

$\vec{P} = 398\hat{i} - 334\hat{j} + 300\hat{k} \text{ lb}$

b) Find:  $\vec{Q} = Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k} = 200 \hat{\lambda}_{AC}$   $\hat{\lambda}_{AC} = \frac{4\hat{j} + 3\hat{k}}{\sqrt{4^2 + 3^2}} = 0.8\hat{j} + 0.6\hat{k}$   
 $= 200 \text{ lb} (0.8\hat{j} + 0.6\hat{k}) = 160\hat{j} + 120\hat{k} \text{ lb}$

c) Find  $|\vec{S}| = \sqrt{144^2 + 60^2 + 156^2} \text{ lb} = 220.6 \text{ lb}$

$\vec{P} = 398\hat{i} - 334\hat{j} + 300\hat{k} \text{ lb}$

$\vec{Q} = 160\hat{j} + 120\hat{k} \text{ lb}$

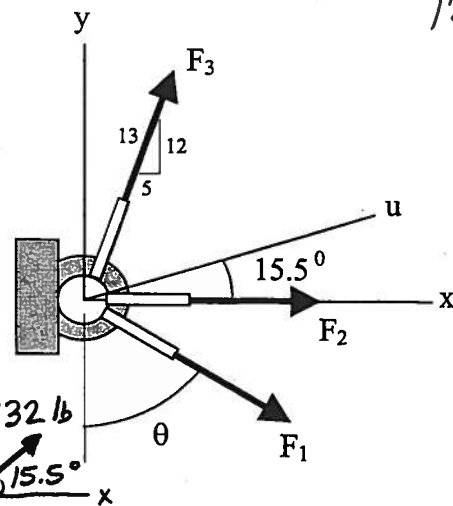
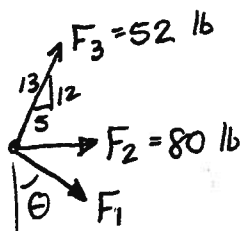
$|\vec{S}| = 221 \text{ lb}$

Three forces act on the bracket as shown where the magnitudes of  $F_2$  and  $F_3$  are known to be 80 lb and 52 lb respectively. Determine the magnitude  $F_1$  and direction  $\theta$  so that the resultant force is directed along the positive  $u$  axis and has a magnitude of 132 lb.

Given:  $F_2 = 80 \text{ lb}$ ,  $F_3 = 52 \text{ lb}$

$$\bar{R} = 132 \text{ lb } \hat{\lambda}_u$$

Find:  $F_1$ ,  $\theta$



$$\Sigma \bar{F} = \bar{R}$$

$$F_1 \sin \theta \hat{i} - F_1 \cos \theta \hat{j} + 80 \hat{i} + \frac{5}{13}(52) \hat{i} + \frac{12}{13}(52) \hat{j} = 132 \cos 15.5 \hat{i} + 132 \sin 15.5 \hat{j}$$

$$i: F_1 \sin \theta + 100 = 127.2 \rightarrow F_1 \sin \theta = 27.2 \text{ lb}$$

$$j: -F_1 \cos \theta + 48 = 35.3 \rightarrow F_1 \cos \theta = 12.7 \text{ lb}$$

$$\frac{F_1 \sin \theta}{F_1 \cos \theta} = \frac{27.2 \text{ lb}}{12.7 \text{ lb}}$$

$$\tan \theta = 2.138 \quad \theta = 64.9^\circ$$

$$F_1 \sin 64.9 = 27.2 \text{ lb}$$

$$F_1 = 30.0 \text{ lb}$$

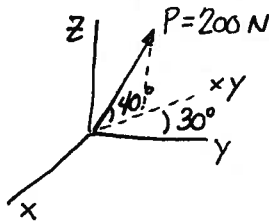
$$F_1 = 30.0 \text{ lb}, \quad \theta = 64.9^\circ$$

Given magnitudes  $P = 200 \text{ N}$  and  $v = 300 \text{ m/s}$  directed as shown in the figure

- Find the rectangular components of  $\mathbf{P}$ .
- Find the rectangular components of  $\mathbf{v}$ .
- Find the component of force vector  $\mathbf{P}$  that acts in the direction of velocity vector  $\mathbf{v}$ .

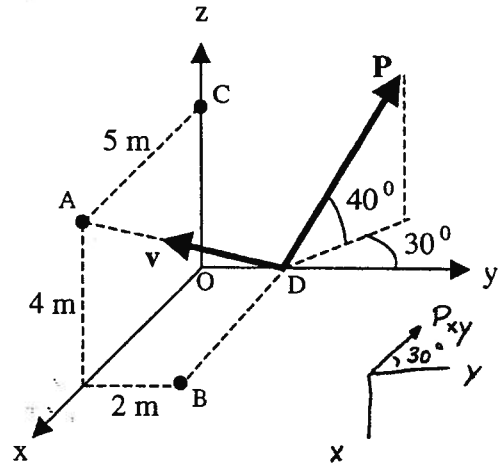
Given:  $P = 200 \text{ N}$   
 $v = 300 \text{ m/s}$

a) Find:  $\bar{\mathbf{P}} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$



$$P_z = 200 \sin 40^\circ \text{ N} \\ = 128.56 \text{ N } (+\hat{k})$$

$$P_{xy} = 200 \cos 40^\circ \text{ N} \\ = 153.21 \text{ N}$$



$$P_x = P_{xy} \sin 30^\circ = 76.60 (-\hat{i}) \text{ N} \\ P_y = P_{xy} \cos 30^\circ = 132.68 \text{ N } (+\hat{j})$$

$$\bar{\mathbf{P}} = -76.6 \hat{i} + 132.7 \hat{j} + 128.6 \hat{k} \text{ N}$$

b) Find  $\bar{\mathbf{v}}$

$$\hat{\lambda}_{DA} = \frac{5\hat{i} - 2\hat{j} + 4\hat{k} \text{ m}}{\sqrt{5^2 + 2^2 + 4^2} \text{ m}} = \frac{5}{\sqrt{45}} \hat{i} - \frac{2}{\sqrt{45}} \hat{j} + \frac{4}{\sqrt{45}} \hat{k} = 0.745 \hat{i} - 0.298 \hat{j} + 0.596 \hat{k}$$

$$\bar{\mathbf{v}} = 300 \text{ m/s } \hat{\lambda}_{DA} = 223.6 \hat{i} - 89.44 \hat{j} + 178.89 \hat{k} \text{ m/s}$$

c) Find  $P_v$

$$P_v = \bar{\mathbf{P}} \cdot \hat{\lambda}_{DA} = (-76.6 \hat{i} + 132.7 \hat{j} + 128.6 \hat{k}) \cdot \left( \frac{5}{\sqrt{45}} \hat{i} - \frac{2}{\sqrt{45}} \hat{j} + \frac{4}{\sqrt{45}} \hat{k} \right) \text{ N}$$

$$= (-76.6) \left( \frac{5}{\sqrt{45}} \right) + (132.7) \left( \frac{-2}{\sqrt{45}} \right) + (128.6) \left( \frac{4}{\sqrt{45}} \right) = -19.98 \text{ N}$$

$$(a) \bar{\mathbf{P}} = -76.6 \hat{i} + 132.7 \hat{j} + 128.6 \hat{k} \text{ N}$$

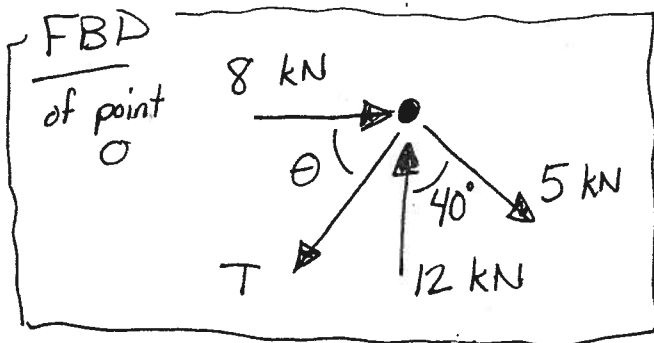
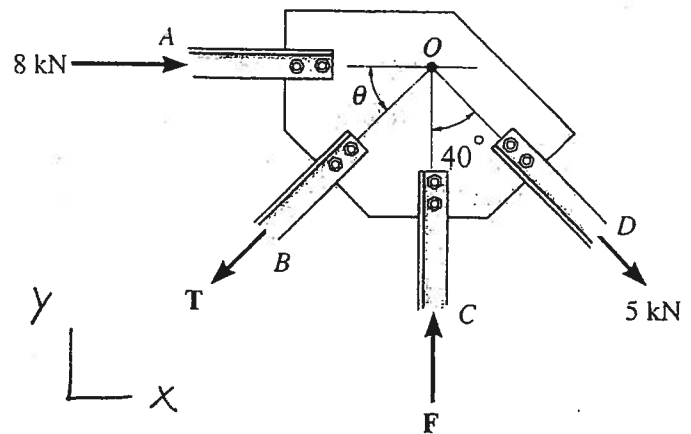
$$(b) \bar{\mathbf{v}} = 223.6 \hat{i} - 89.4 \hat{j} + 178.9 \hat{k} \text{ m/s}$$

$$(c) P_v = -19.98 \text{ N}$$

The gusset plate is subjected to the forces of four members as shown. If the magnitude of  $F$  is 12 kN, determine the magnitude  $T$  of the force in member B and its proper orientation  $\theta$  for equilibrium. The forces are concurrent at point O.

GIVEN:  $F = 12 \text{ kN}$

FIND:  $T, \theta$  for equilibrium



For equilibrium  $\Sigma \vec{F} = 0$  or in 2-D  $\Sigma F_x = 0$  and  $\Sigma F_y = 0$

$$\rightarrow \Sigma F_x = 0 = 8 - T \cos \theta + 5 \sin 40 \rightarrow T \cos \theta = 11.21 \text{ kN} \quad (1)$$

$$\uparrow \Sigma F_y = 0 = 12 - T \sin \theta - 5 \cos 40 \rightarrow T \sin \theta = 8.17 \text{ kN} \quad (2)$$

divide (2) by (1)  $\frac{T \sin \theta}{T \cos \theta} = \frac{8.17 \text{ kN}}{11.21 \text{ kN}}$

$$\tan \theta = 0.729 \quad \theta = 36.1^\circ$$

Using (1)  $T \cos 36.1 = 11.21 \text{ kN}$   
 $T = 13.9 \text{ kN}$

check in (2)  $13.9 \sin \theta = 8.17 \quad \theta = 36.0^\circ$   
OK

$$T = 13.9 \text{ kN}$$

$$\theta = 36.1^\circ$$

The pillar shown supports a vertical 30 N force at F. The bottom of the pillar is supported at C by 2 rods and 2 cables as shown. The tension in cable CA is 160 N and the tension in cable CB is 90 N.

Neglecting the weights of the pillar, rods and cables, determine the magnitude of force in members CD and CE required to maintain static equilibrium.

Given:  $F_{CA} = 160 \text{ N}$ ,  $F_{CB} = 90 \text{ N}$ , system in equilb, FIND:  $F_{CD}$ ,  $F_{CE}$

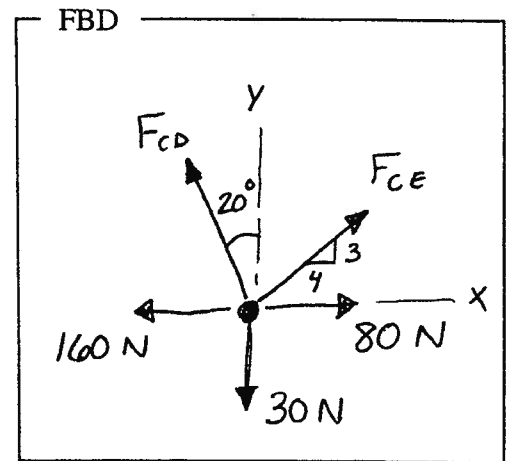
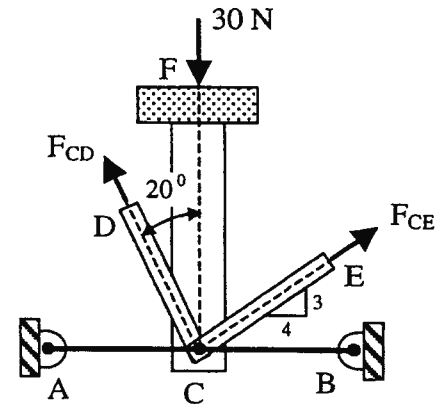
Equilibrium  $\Sigma \vec{F} = 0$

$$\rightarrow \Sigma F_x = 0 = 90 - 160 + F_{CE} \left(\frac{4}{5}\right) - F_{CD} \sin 20$$

$$\frac{4}{5} F_{CE} - F_{CD} \sin 20 = 70 \text{ N}$$

$$+\uparrow \Sigma F_y = 0 = F_{CD} \cos 20 + F_{CE} \left(\frac{3}{5}\right) - 30$$

$$\frac{3}{5} F_{CE} + F_{CD} \cos 20 = 30 \text{ N}$$



2 equations / 2 unknowns

$$F_{CE} = \frac{5}{3} (30 \text{ N} - F_{CD} \cos 20) = 50 \text{ N} - \frac{5}{3} F_{CD} \cos 20$$

$$\frac{4}{5} \left( 50 \text{ N} - \frac{5}{3} F_{CD} \cos 20 \right) - F_{CD} \sin 20 = 70 \text{ N}$$

$$F_{CD} \left( -\frac{4}{3} \cos 20 - \sin 20 \right) = 30 \text{ N}$$

$$F_{CD} = \frac{30 \text{ N}}{-1.5949} = -18.81 \text{ N}$$

$$F_{CE} = 50 \text{ N} - \frac{5}{3} (-18.81 \text{ N}) \cos 20$$

$$F_{CE} = 79.5 \text{ N}$$

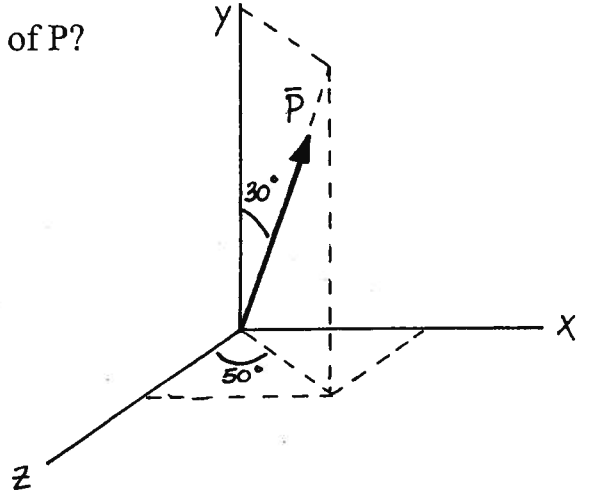
$$F_{CD} = -18.81 \text{ N}$$

$$F_{CE} = 79.5 \text{ N}$$

4.1 Which equation describes the x component of P?

circle one

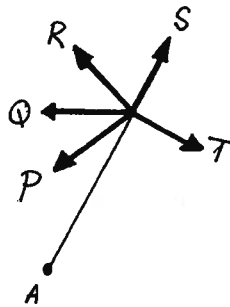
- a)  ~~$P \cos(30)$~~       e)  ~~$P \cos(30) \cos(50)$~~   
 b)  ~~$P \sin(30)$~~       f)  $P \sin(30) \cos(50)$   
 c)  ~~$P \cos(50)$~~       g)  ~~$P \cos(30) \sin(50)$~~   
 d)  ~~$P \sin(50)$~~       h)  $P \sin(30) \sin(50)$



4.2 Which force does **NOT** produce a moment about point A?

circle one

- P  
 Q  
 R  
 S  
 T



4.3 Which vector below is **NOT** a unit vector?

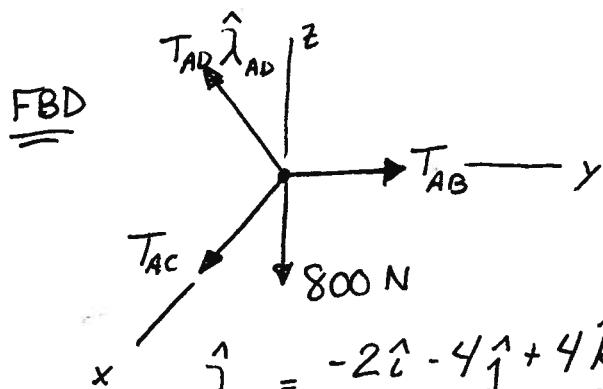
circle one

- a)  $0.342 \mathbf{i} - 0.587 \mathbf{j} + 0.734 \mathbf{k}$   
 b)  $-0.911 \mathbf{i} + 0.135 \mathbf{j} + 0.390 \mathbf{k}$   
 c)  $0.459 \mathbf{i} + \underline{\underline{1.321 \mathbf{j}}} - 0.198 \mathbf{k}$   
 d)  $0.282 \mathbf{i} + 0.681 \mathbf{j} + 0.676 \mathbf{k}$   
 a)  $0.586 \mathbf{i} + 0.575 \mathbf{j} + 0.571 \mathbf{k}$

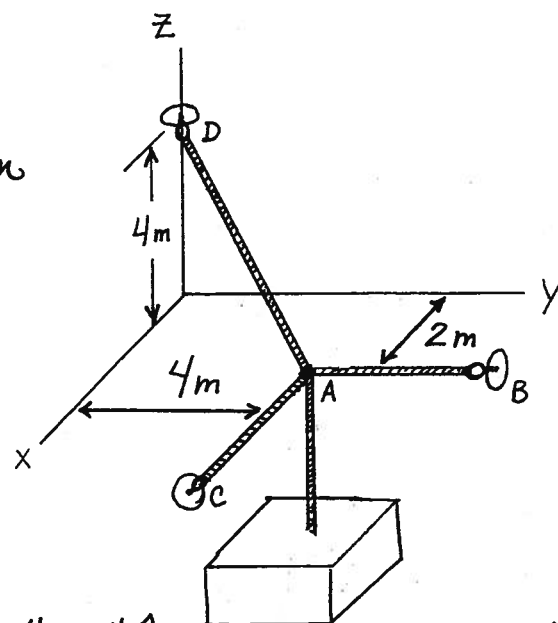
Three cables are used to support the 800 N box. Determine the force developed in each cable for equilibrium.

Given:  $W = 800 \text{ N}$ , system in equilibrium

Find:  $T_{AB}$ ,  $T_{AC}$ ,  $T_{AD}$



$$\hat{\lambda}_{AD} = \frac{-2\hat{i} - 4\hat{j} + 4\hat{k}}{\sqrt{2^2 + 4^2 + 4^2}} = \frac{-2}{6}\hat{i} - \frac{4}{6}\hat{j} + \frac{4}{6}\hat{k} = -\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$



$$\Sigma \vec{F} = 0 = T_{AB}\hat{j} + T_{AC}\hat{i} + T_{AD}\left(-\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right) - 800\hat{k} \text{ N} = 0$$

$$i: T_{AC} - \frac{1}{3}T_{AD} = 0$$

$$j: T_{AB} - \frac{2}{3}T_{AD} = 0$$

$$k: \frac{2}{3}T_{AD} - 800 = 0 \rightarrow T_{AD} = \frac{3}{2}(800) = 1200 \text{ N}$$

$$T_{AC} = \frac{1}{3}T_{AD} = \frac{1}{3}(1200) = 400 \text{ N}$$

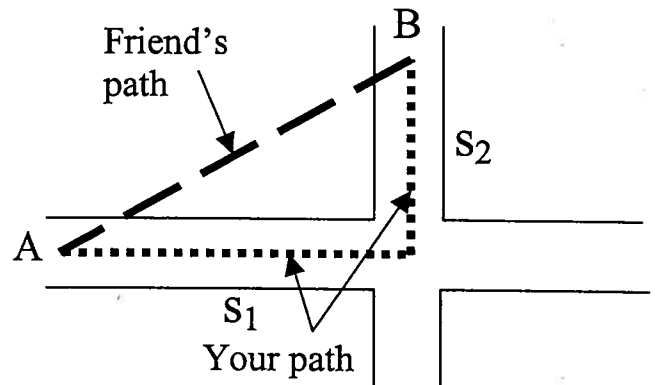
$$T_{AB} = \frac{2}{3}T_{AD} = \frac{2}{3}(1200) = 800 \text{ N}$$

$$T_{AB} = 800 \text{ N}, T_{AC} = 400 \text{ N}$$

$$T_{AD} = 1200 \text{ N}$$

You walk from point A to point B along level sidewalks of length  $s_1$  and  $s_2$  and your friend takes the straight path through a level field. The distance your friend walks is:

- A) equal to the arithmetic sum of  $s_1$  and  $s_2$  (i.e. the distance you walk)
- B) greater than the arithmetic sum of  $s_1$  and  $s_2$
- C) greater than either  $s_1$  or  $s_2$  but less than the arithmetic sum of  $s_1$  and  $s_2$
- D) greater than the vector sum of  $s_1$  and  $s_2$



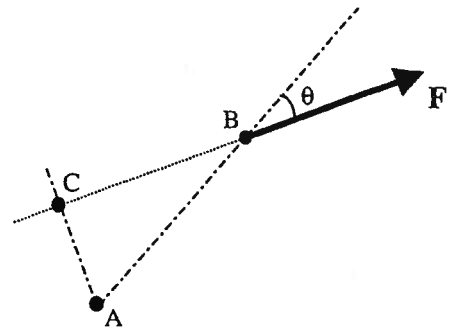
Which of the following is NOT an external effect?

- A) Deformation
- B) Motion
- C) State of rest
- D) Support reactions

When dealing only with EXTERNAL effects on a rigid body, a force may be treated as what kind of vector?

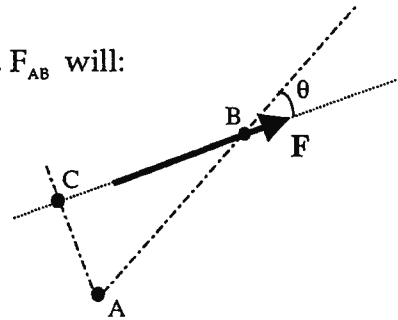
- E) Equivalent vector
- F) Fixed vector
- G) Free vector
- H) Sliding vector

For questions a) to d) indicate whether the magnitude of the component of force  $F$  along line  $AB$  ( $F_{AB}$ ) will increase, decrease or stay the same. Circle the correct response.



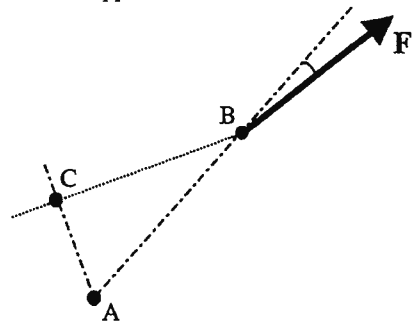
a) If the point of application of the force moves from B to C,  $F_{AB}$  will:

Increase      Decrease      Stay the same



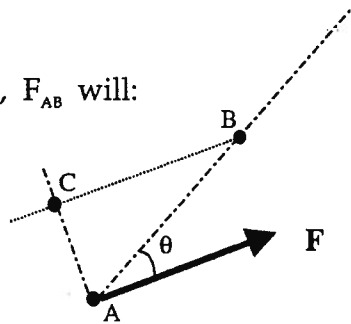
b) If the angle  $\theta$  between  $F$  and line  $AB$  decreases,  $F_{AB}$  will:

Increase      Decrease      Stay the same



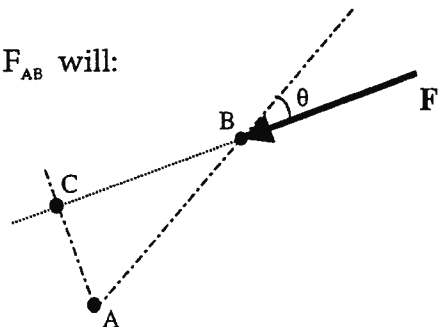
c) If the point of application of the force moves from B to A,  $F_{AB}$  will:

Increase      Decrease      Stay the same



d) If the force is opposite in direction, i.e. points toward C,  $F_{AB}$  will:

Increase      Decrease      Stay the same



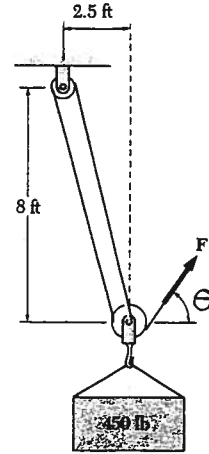
Read

A 450 lb crate is supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force  $F$  that should be exerted on the free end of the rope to maintain static equilibrium.

Assume the pulleys are frictionless and each pulley diameter is negligible.

Given:  $W = 450$  lb

FIND:  $F, \theta$  for equilibrium



Apply  $\Sigma F_x = 0$  } 2 eq., 2 unknowns  $F, \theta$   
 $\Sigma F_y = 0$

can get  $\beta$  from geometry

Solve

$$\rightarrow \Sigma F_x = 0 = F \cos \theta - 2F \cos \beta$$

$$\tan \beta = \frac{8}{2.5} \quad \beta = 72.6^\circ$$

$$F \cos \theta = 0.597 F$$

$$\cos \theta = 0.597$$

$$\theta = 53.4^\circ$$

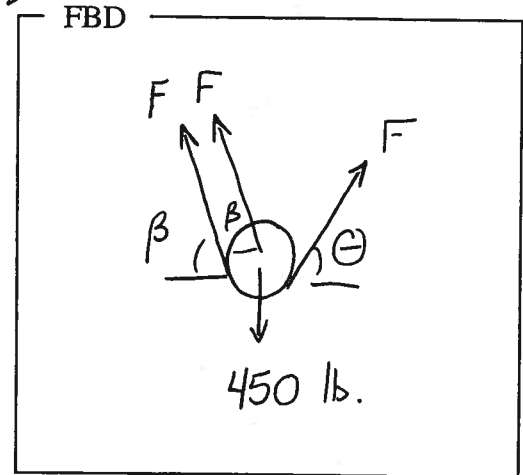
$$\uparrow \Sigma F_y = 0 = 2F \sin 72.6 + F \sin 53.4 - 450 = 0$$

$$2.711 F = 450$$

$$F = 166 \text{ lb}$$

check seems reasonable magnitude

Draw



$$|F| = \underline{166} \text{ lb}$$

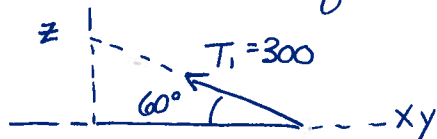
$$\theta = \underline{53.4^\circ} \text{ degrees}$$

The magnitude of  $T_1 = 300$  N.

Using the geometry as shown in the figure

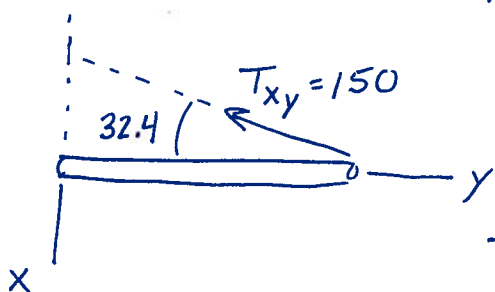
- Find the rectangular components of  $T_1$ .
- Knowing that the resultant of the 3 forces at B is a vector directed along the y axis from B to A, find the magnitude of the tension  $T_2$  and the resultant R.

a) Find  $T_1$  vector equation



$$T_z = 300 \sin 60 = 260 \text{ N}$$

$$T_{xy} = 300 \cos 60 = 150 \text{ N}$$



$$T_x = T_{xy} \sin 32.4 = 150 \sin 32.4 = 80.4$$

(negative x direction)

$$T_y = T_{xy} \cos 32.4 = 126.6 \text{ (negative y dir.)}$$

$$\vec{T}_1 = -80.4 \hat{i} - 126.6 \hat{j} + 260 \hat{k} \text{ N}$$

$$\text{check } \sqrt{80.4^2 + 126.6^2 + 260^2} = 300 \checkmark$$

b) Given:  $\vec{R} = -R \hat{j}$

Find:  $T_2, R$

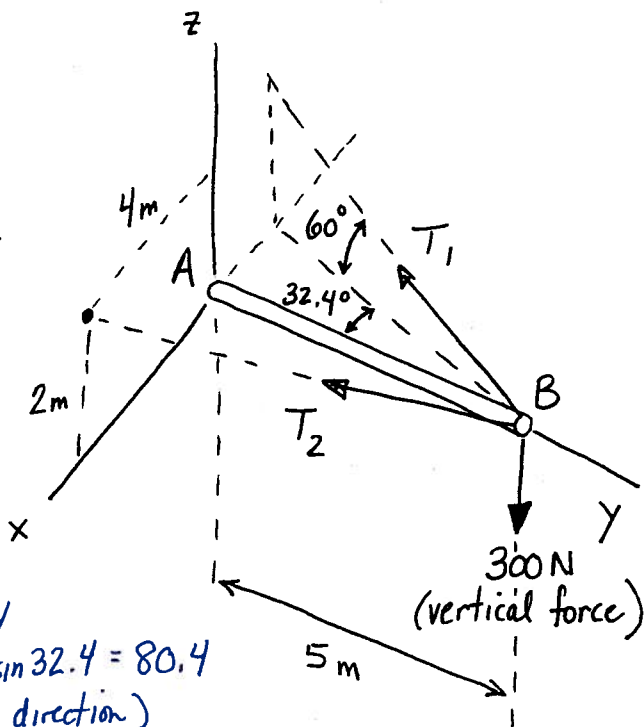
$$\vec{T}_2 = T_2 \hat{u}_2 = T_2 \left( \frac{4\hat{i} - 5\hat{j} + 2\hat{k}}{\sqrt{4^2 + 5^2 + 2^2}} \right) = T_2 (0.596 \hat{i} - 0.745 \hat{j} + 0.298 \hat{k})$$

$$\vec{R} = \vec{T}_1 + \vec{T}_2 \Rightarrow -R \hat{j} = -80.4 \hat{i} - 126.6 \hat{j} + 260 \hat{k} + 0.596 T_2 \hat{i} - 0.745 T_2 \hat{j} + 0.298 T_2 \hat{k} - 300 \hat{k}$$

$$\hat{i}: 0 = -80.4 + 0.596 T_2 \quad T_2 = 134.9 \text{ N}$$

$$\hat{j}: -R = -126.6 - 0.745(134.9) \quad R = 227 \text{ N}$$

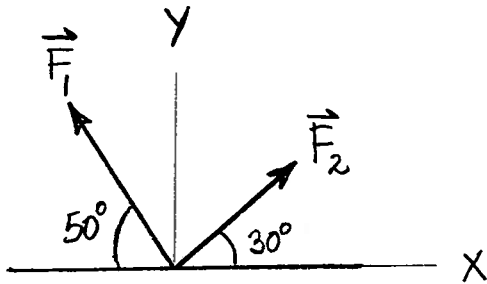
$$\text{check } \hat{k}: 0 = 260 + 0.298 T_2 - 300 \quad T_2 = 134.3 \checkmark$$



$$(a) \vec{T}_1 = \underline{-80.4} \hat{i} \underline{-126.6} \hat{j} \underline{+260} \hat{k} \text{ N}$$

$$(b) |T_2| = \underline{135} \text{ N} \quad |R| = \underline{227} \text{ N}$$

a.) Knowing that the resultant of  $\vec{F}_1$  and  $\vec{F}_2$  is a vertical force of 800 N, find the magnitudes of  $\vec{F}_1$  and  $\vec{F}_2$



Given:  $R = 800 \hat{j}$   
 Find:  $|F_1|, |F_2|$

$$\vec{R} = \sum \vec{F}$$

$$800 \hat{j} = F_1 (-\cos 50 \hat{i} + \sin 50 \hat{j}) + F_2 (\cos 30 \hat{i} + \sin 30 \hat{j})$$

$$i: 0 = -F_1 \cos 50 + F_2 \cos 30 \quad F_1 = F_2 \frac{\cos 30}{\cos 50} = 1.347 F_2$$

$$j: 800 = F_1 \sin 50 + F_2 \sin 30$$

$$800 = (1.347 F_2) \sin 50 + F_2 \sin 30$$

$$800 = 1.532 F_2 \quad F_2 = 522$$

$$F_1 = 703$$

$$|F_1| = \underline{703 \text{ N}} \quad \text{and} \quad |F_2| = \underline{522 \text{ N}}$$

b.) Find the component of  $F_1$  in the direction of  $F_2$

Find:  $\vec{F}_1 \cdot \hat{u}_2$

$$\vec{F}_1 = 703 (-\cos 50 \hat{i} + \sin 50 \hat{j}) = -452 \hat{i} + 538.5 \hat{j}$$

$$\hat{u}_2 = \cos 30 \hat{i} + \sin 30 \hat{j} = 0.866 \hat{i} + 0.5 \hat{j}$$

$$\vec{F}_1 \cdot \hat{u}_2 = (-452)(0.866) + (538.5)(0.5)$$

$$= -122 \text{ N}$$

$$\boxed{F_1}_{2 \text{ dir.}} = 122 \text{ N}$$