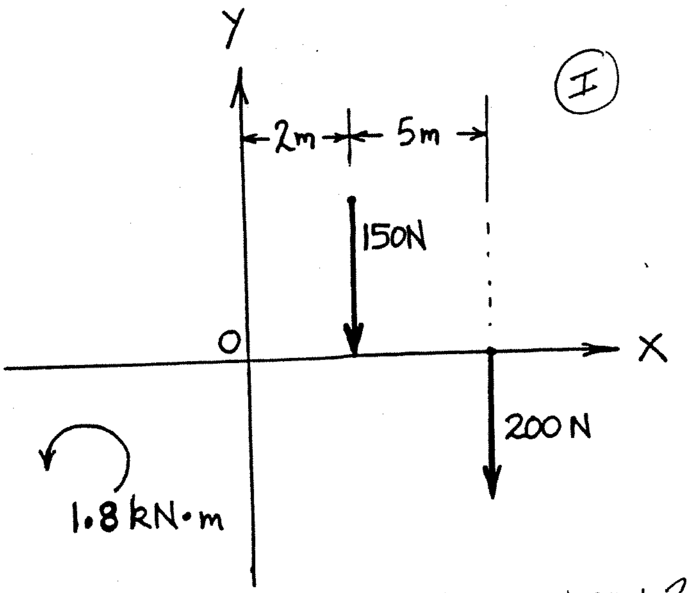
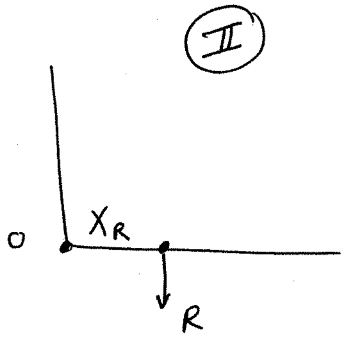
For the parallel, coplanar forces and couple system shown : find the simplest resultant (i.e. single force) and its location.

$\boxed{\text{Find: } R,\ X_R}$





$$\downarrow \ R = \Sigma F = 150 + 200 = 350 \text{ N}$$

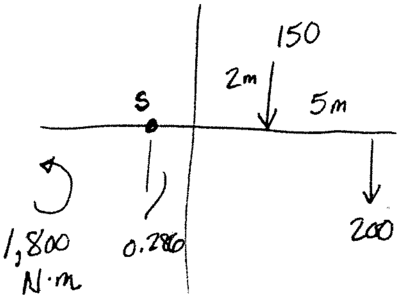
$$\overset{\curvearrowleft}{+} \quad \Sigma M_O^{I} = \Sigma M_O^{II}$$

$$1{,}800 \text{ N·m} - 150(2) - 200(7) = -350(X_R)$$

$$100 = -350\, X_R$$

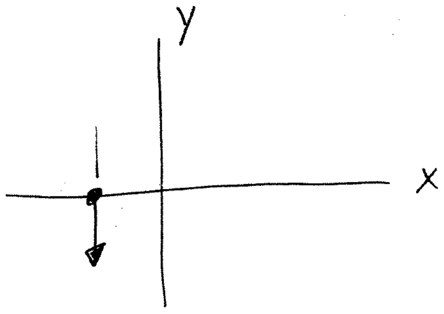
$$X_R = -0{,}286$$

Check $\Sigma M_S = 0$?

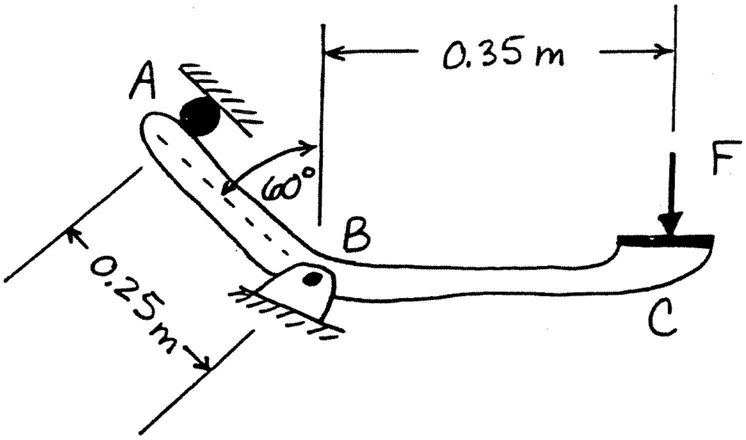


$$\overset{\curvearrowleft}{+} \ \Sigma M_S = 1800 - 150(2.286) - 200(7.286) = 0$$

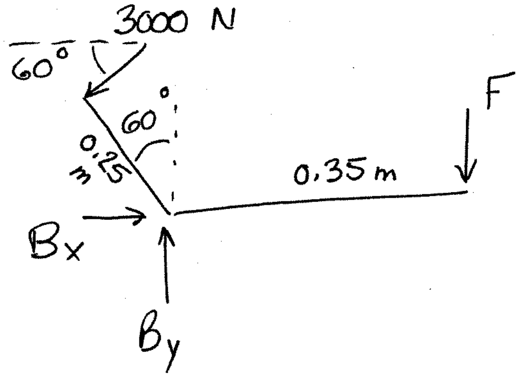
yes!



A vertical force F is applied to the solid bracket ABC at point C as shown. The bracket is supported by a roller at A and a pin at B. Given that the roller will break when the total magnitude of the roller force exceeds 3000 N, find the maximum value of F that can be applied and the magnitude of the pin reaction at B when F is at its maximum value.



FBD



Given: $R_A = 3000\ N$

FIND: $F, |B|$

PLAN $\Sigma M_B \rightarrow F$, $\Sigma F_x \rightarrow B_x$, $\Sigma F_y \rightarrow B_y$

$\overset{+}{\circlearrowleft}\ \Sigma M_B = 3000(0.25) - F(0.35)$ $\quad F = 2143\ N$

$\rightarrow \Sigma F_x = 0 = B_x - 3000 \cos 60$ $\quad B_x = 1500\ N$

$\uparrow \Sigma F_y = 0 = B_y - 3000 \sin 60 \; - \underset{2143}{\underset{\uparrow}{F}}$ $\quad B_y = 4741\ N$

$|B| = \sqrt{1500^2 + 4741^2}$

$= 4973\ N$

$$\boxed{\begin{aligned} F &= 2143\ N \\ |B| &= 4973\ N \end{aligned}}$$

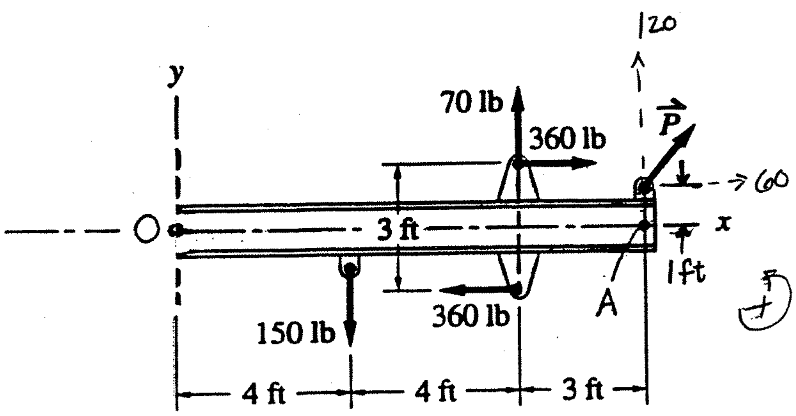
For the beam OA with the force-couple system shown with $\vec{P} = 60\vec{i} + 120\vec{j}$ (lb), determine the simplest resultant $\vec{R}$ and the position along the beam that it intersects.

Given: $\bar{P} = 60\hat{i} + 120\hat{j}$ lb

Find: $\bar{R}, X_R$

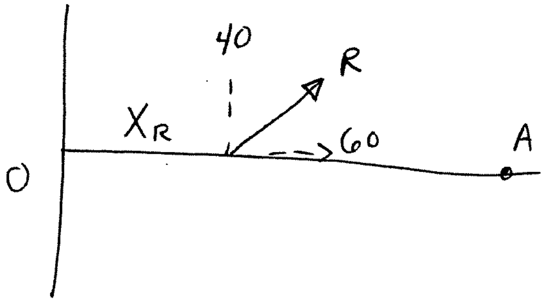
ANSWER

$\vec{R} =$ $60\hat{i} + 40\hat{j}$ lb @ x = 3.5 ft



$$\bar{R} = -150\hat{j} + 70\hat{j} + 60\hat{i} + 120\hat{j} \text{ lb}$$
$$= 60\hat{i} + 40\hat{j} \text{ lb.}$$

$$\circlearrowleft + \ \Sigma M_O = 70(8) - 150(4) + 120(11) - 60(1) - 360(3) = 140 \text{ lb·ft.}$$



$$\circlearrowleft \ \Sigma M_O = 40 X_R$$

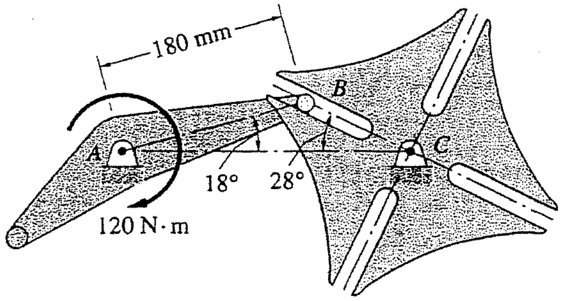
$$140 = 40 X_R, \ X_R = 3.5 \text{ ft.}$$

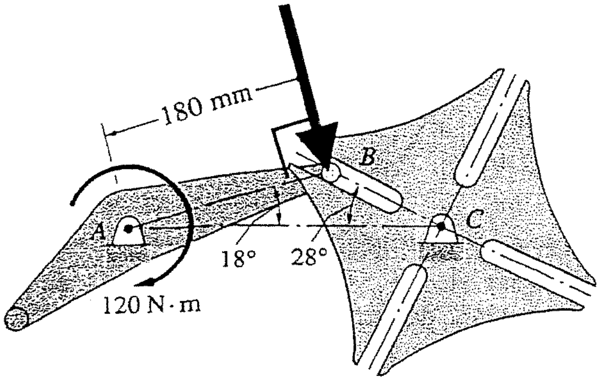
Check $\circlearrowleft + \ \Sigma M_A = 150(7) - 70(3) - 360(3) - 60 = 40(11 - X_R)$

$$-300 = -40(11 - X_R)$$

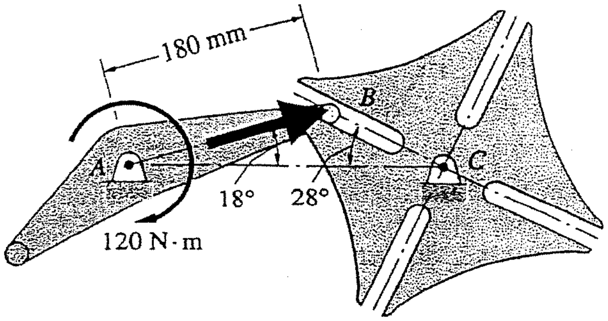
$$7.5 = 11 - X_R \quad X_R = 3.5 \quad \text{ok.}$$

Neglecting friction in the mechanism shown, identify the correct direction for the slider pin reaction at B.

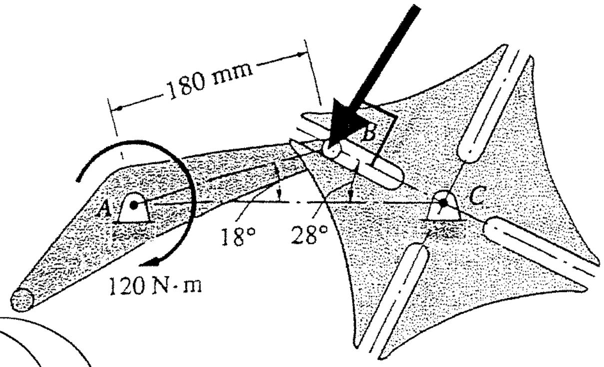




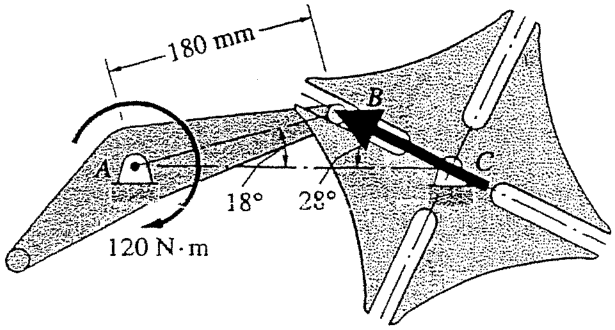
a) perpendicular to line AB



b) along line AB



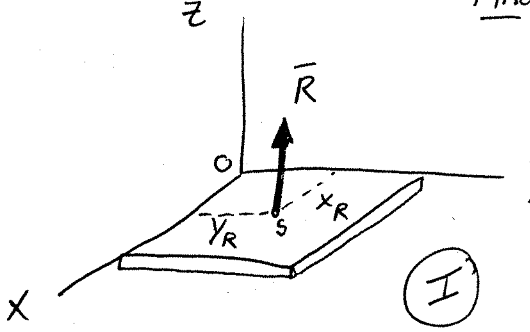
[✓ c) perpendicular to line BC]

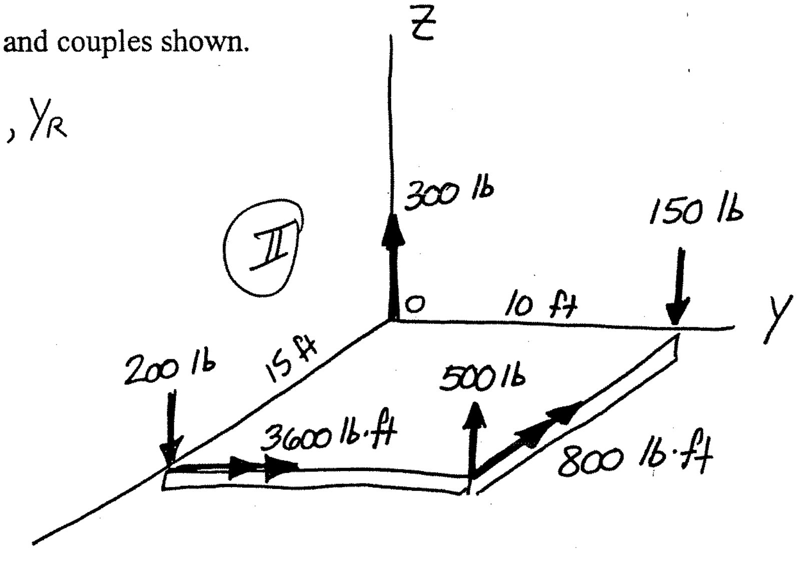


d) along line BC

Find the resultant for the system of forces and couples shown.

Find: $\bar{R}, X_R, Y_R$

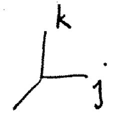




$\bar{R} = \Sigma F$

$\bar{R} = 300\hat{k} - 200\hat{k} - 150\hat{k} + 500\hat{k} \;= 450\hat{k}$ lb.

$\Sigma M_o^{I} = \Sigma M_o^{II}$;



$(X_R\hat{i} + Y_R\hat{j}) \times 450\hat{k} = (15\hat{i}) \times (-200\hat{k}) + (10\hat{j}) \times (-150\hat{k}) + (15\hat{i} + 10\hat{j}) \times 500\hat{k} + 3600\hat{j} - 800\hat{i}$ lb·ft

$-450X_R\hat{j} + 450Y_R\hat{i} = -3000(-\hat{j}) - 1500(\hat{i}) + 7500(-\hat{j}) + 5000(\hat{i}) + 3600\hat{j} - 800\hat{i}$

i: $450\,Y_R = -1500 + 5000 - 800$ $\quad Y_R = 6$ ft.

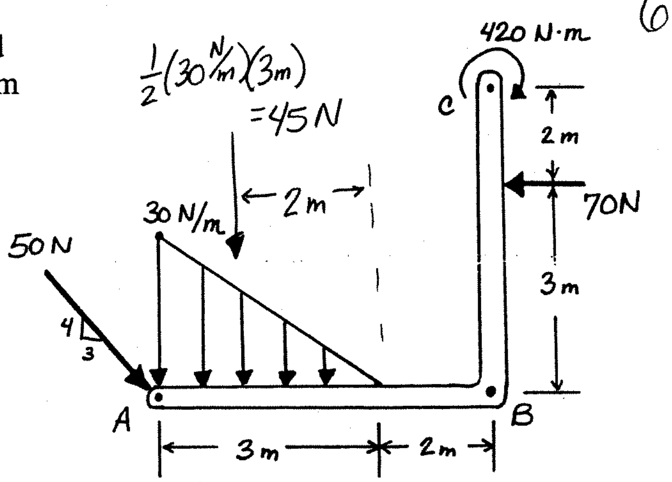
j: $-450\,X_R = 3000 - 7500 + 3600 = -900$ $\quad X_R = 2$ ft.

Check by taking $\Sigma\bar{M}_s = 0$?
yes!

$$\boxed{\begin{array}{l}\bar{R} = 450\hat{k} \text{ lb} \\ X_R = 2 \text{ ft} \quad Y_R = 6 \text{ ft.}\end{array}}$$

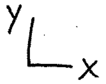
The rigid member ABC is subjected to two concentrated forces, a distributed load that varies linearly from 30 N/m to zero and a 420 N-m couple as shown.

(a) Calculate the resultant force and couple acting at B. Sketch these on the blank figure given below.



Find: $\bar{R} = \Sigma \bar{F}$

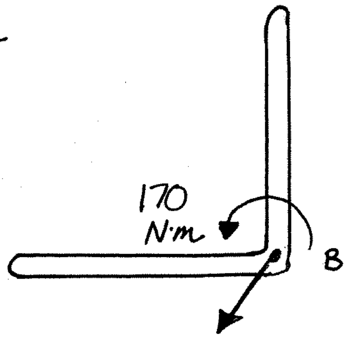
$C_R = \Sigma M_B$



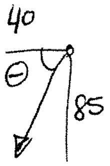
$\bar{R} = \frac{3}{5}(50)\hat{i} - \frac{4}{5}(50)\hat{j} - 45\hat{j} - 70\hat{i}$

$= -40\hat{i} - 85\hat{j}$ N

$(+\circlearrowleft)\ \Sigma M_B = 70(3) - 420 + 45(4) + \frac{4}{5}(50)(5) = 170$ N·m



$|R| = \sqrt{40^2 + 85^2} = 93.9$ N



$\tan\theta = \frac{85}{40}$ $\theta = 64.8°$

R = 93.9 N

θ = 64.8°

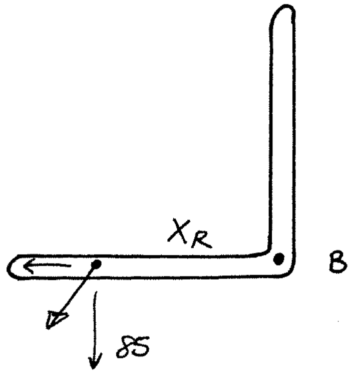
C_R = 170 N·m

(b) Determine the location of the simplest resultant (for only resultant). Sketch the resultant and label this location on the blank figure below.

Find: X_R

$\circlearrowleft\ \Sigma M_B = 170 = 85 X_R$

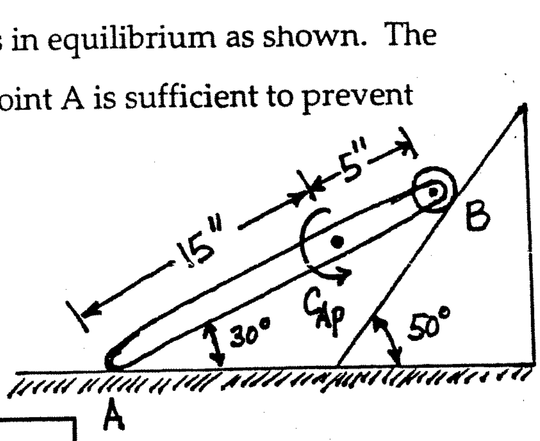
$\boxed{X_R = 2\text{ m}}$

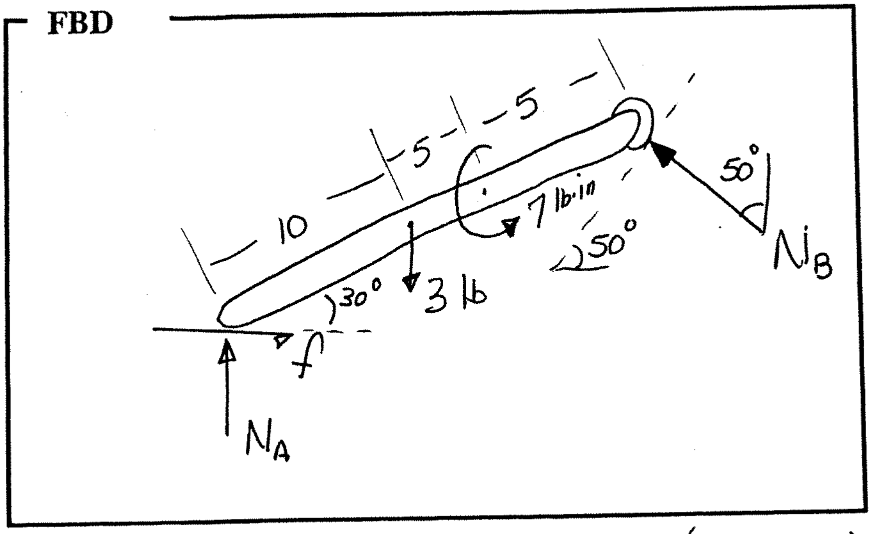


A metal rod AB weighing 3 lb with a roller on one end is in equilibrium as shown. The applied couple (C_{AP}) is 7 lb–in. and the friction force at point A is sufficient to prevent sliding.

a) Sketch the complete free body diagram (FBD).

b) Use this FBD to calculate the reaction forces and their directions at A and B.





Given: $C_{AP} = 7$ in·lb

$W = 3$ lb

friction at A prevents sliding

Find: N_A, f_A, N_B

$$\circlearrowleft \ \Sigma M_A = (N_B \cos 50)(20 \cos 30) + (N_B \sin 50)(20 \sin 30) + 7 - 3(10 \cos 30) = 0$$

$$18.79 \, N_B = 18.98 \qquad N_B = 1.01 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = f - 1.01 \sin 50 = 0. \qquad f = 0.77 \text{ lb}$$

$$+\uparrow \ \Sigma F_y = N_A - 3 + 1.01 \cos 50 = 0 \qquad N_A = +2.35 \text{ lb.}$$

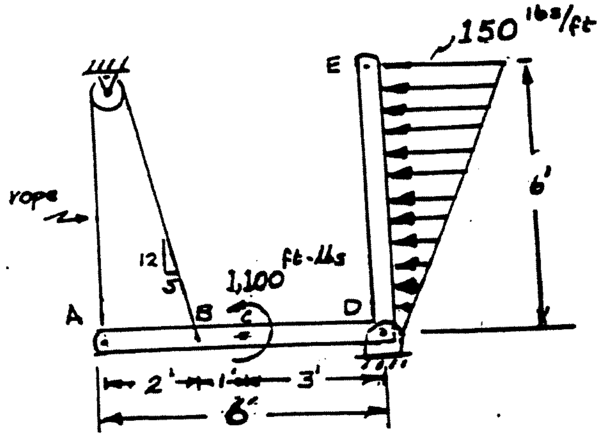
$$\text{check } \circlearrowleft \ \Sigma M_B = 7 + 3(10 \cos 30) + (0.77)(20 \sin 30) - (2.35)(20 \cos 30)$$

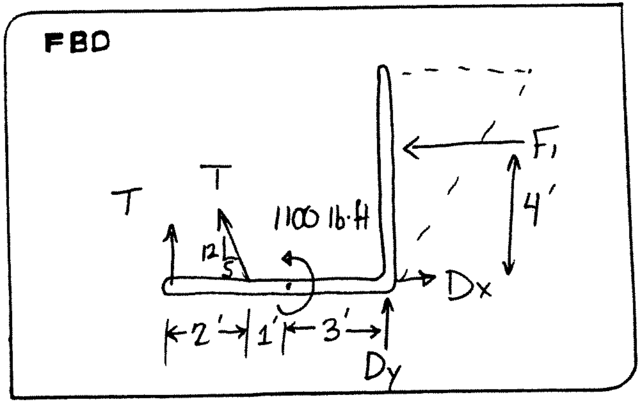
$= -3.48$ ⇒ is this close enough to zero? Check round off.

$N_B = 1.0101$, $f = 0.7738$ $N = 2.3507$ $\Sigma M_B = 0.0034$

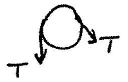
ok

Consider the rigid frame ADE as shown in the sketch to the right. A rope is attached at A, passes over a frictionless pulley and is attached to the frame also at B. A pure couple (1,100 ft-lbs) is applied at point C, and the frame is connected to the ground through a smooth pin at point D. Finally a uniformly increasing load is applied to the vertical section DE rising to an intensity of 150 lbs/ft at E. If the system is in equilibrium, determine the tension T in the rope and the pin force on the frame at D.





Given: System in equilibrium
Pulley is frictionless →



Find: T, $\bar{D}$

$F_1 = \frac{1}{2}(150 \frac{lb}{ft})(6 ft) = 450 \ lb$

$\overset{+}{\circlearrowleft} \Sigma M_D = 450(4) + 1100 - T(6) - \frac{12}{13} T(4) = 0$

$2900 \ lb \cdot ft = 9.69\overset{ft}{2} \ T$

$T = 299.2 \ lb.$

$\boxed{\begin{array}{l} T = \underline{299.2 \ lb.} \\ \vec{D} = \underline{565.1 \hat{i} - 575.4 \hat{j}} \ lb \end{array}}$

$\rightarrow \Sigma F_x = Dx - 450 - \frac{5}{13}(299.2) = 0$

$Dx = 565.1 \ lb.$

$\uparrow \Sigma F_y = (299.2) + \frac{12}{13}(299.2) + D_y = 0$

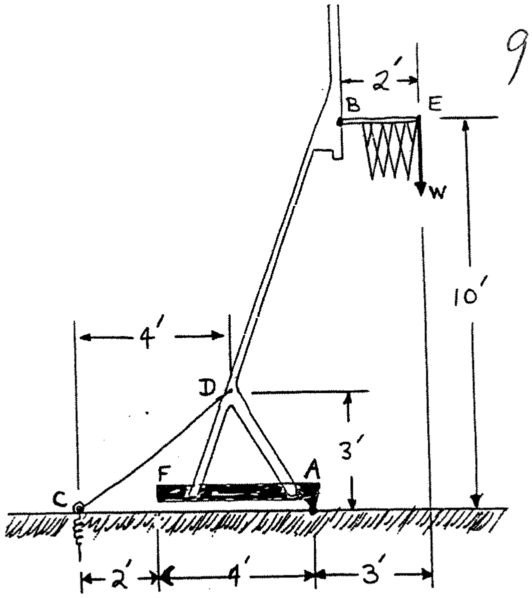
$D_y = -575.4 \ lb.$

Check $\overset{+}{\circlearrowleft} \Sigma M_A = \frac{12}{13} T(2) + 1100 + (-575.4)(6) + 450(4) = 0$

$T = 299.2$ $\underline{\underline{ok}}$

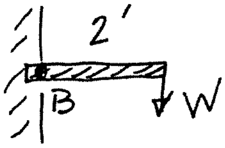
A portable basketball hoop has a homogenous base AF that weighs 195 lbs. The base is anchored by a pin at A and a rope CD. Neglect the weight of the rim, backboard, and support structure.

a) If the 'break-away' rim can withstand a 392 ft-lb couple at point B, what is the weight W of the heaviest player that can hang on the rim at point E without 'breaking' the rim?



Given: $M_B)_{max} = 392$ ft·lb

Find: W



$$M_B = \overset{ft}{2W} \leq \overset{ft\cdot lb}{392}$$

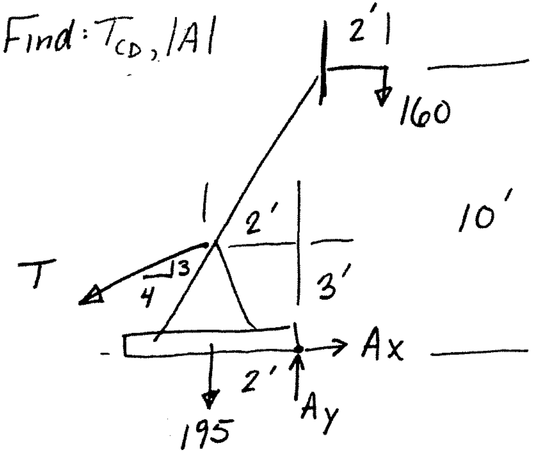
$W \leq 196$ lb.

W max = 196 lb.

b) Find the tension in the rope CD and the magnitude of the pin reaction at A if a player weighing W = 160 lb hangs on the rim at point E.

Given: W = 160 lb

Find: T_{CD}, $|A|$



$$\circlearrowleft + \sum M_A = \frac{4}{5}T(3) + \frac{3}{5}T(2) + 195(2) - 160(3) =$$

$$\frac{18}{5}T = 90 \qquad T = 25 \text{ lb}$$

$$\rightarrow \sum F_x = A_x - \frac{4}{5}(25) = 0$$

$$A_x = 20$$

$$\uparrow \sum F_y = A_y - 160 - 195 - \frac{3}{5}(25) = 0$$

$$A_y = 370$$

$$|A| = \sqrt{20^2 + 370^2} = 370.5 \text{ lb.}$$

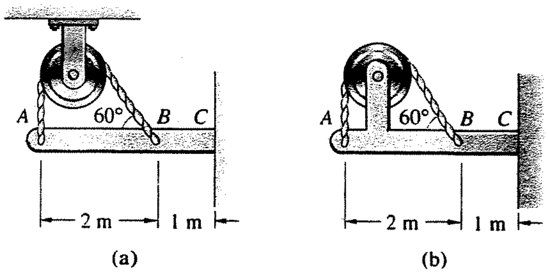
T CD = 25 lb.

|A| = 370.5 lb.

If the tension in the cable is the same for both configurations shown, will the reactions at C be different in (a) and (b) ?

● YES, both figures will produce different wall reactions.

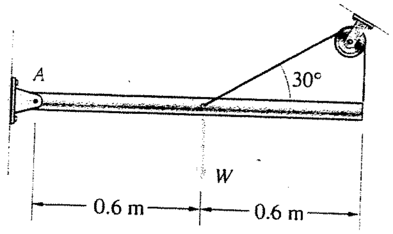
○ NO, the reactions in each of the two cases will be the same.



(a) (b)

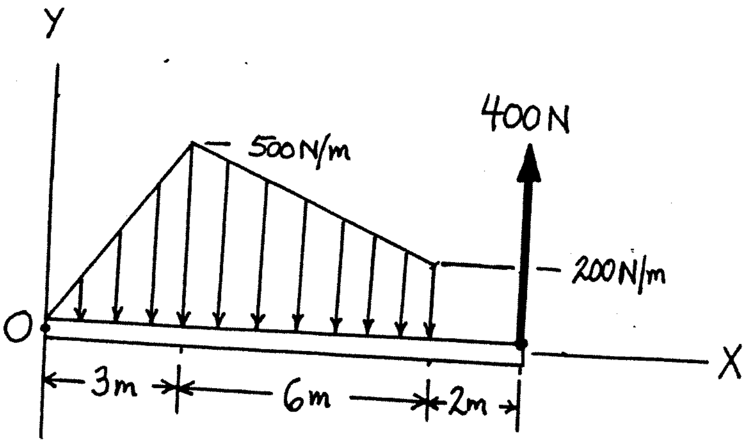
If the weight of this bar is given and the pulley is frictionless, how many independent unknowns will appear in the FBD of this bar?

1
2
[✓ 3]
4
5
6
7
8

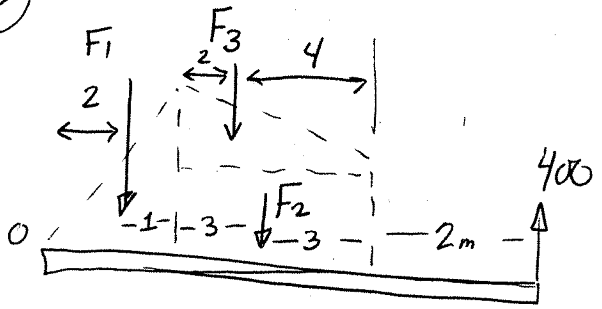


A distributed load force and point force are applied to the beam, as shown. Determine the simplest resultant force and where it acts on the beam.

Find: R, X_R



(I)



$$F_1 = \frac{1}{2}\left(500 \frac{N}{m}\right)(3m) = 750\ N$$

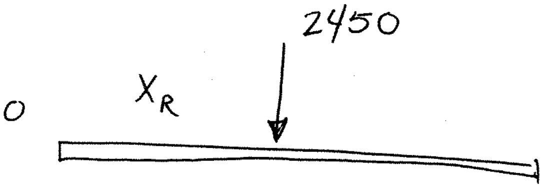
$$F_2 = 200 \frac{N}{m}(6m) = 1200\ N$$

$$F_3 = \frac{1}{2}\left(300 \frac{N}{m}\right)(6m) = 900\ N$$

$$\downarrow \bar{R} = \Sigma F = 750 + 1200 + 900 - 400$$

$$\boxed{\bar{R} = 2450\ N \downarrow}$$

(II)

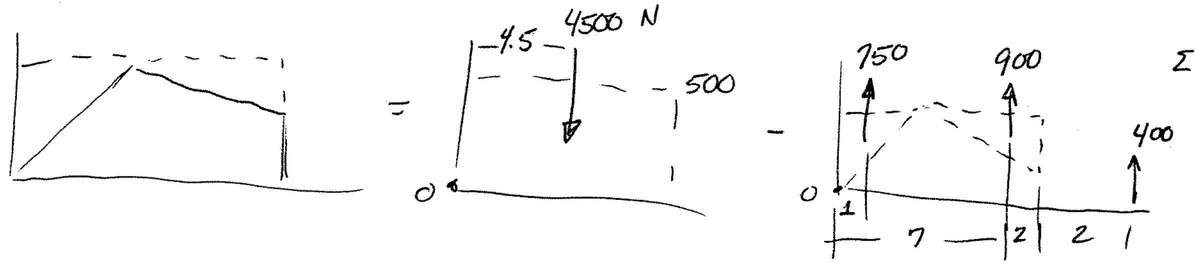


$$\circlearrowleft + \quad \Sigma M_o^{I} = \Sigma M_o^{II}$$

$$750(2) + 900(5) + 1200(6) - 400(11) = +2450 X_R$$

$$8800 = +2450 X_R \qquad \boxed{X_R = +3.60\ m}$$

check



$R = 2450\ N \downarrow$

ΣM_o

$\circlearrowleft = 750(1)$

$+900(7)$

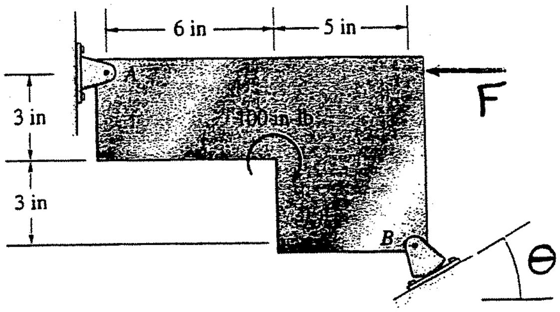
$+400(11)$

$-4500(4.5)$

$= -2450 X_R$

$X_R = 3.60$ ok

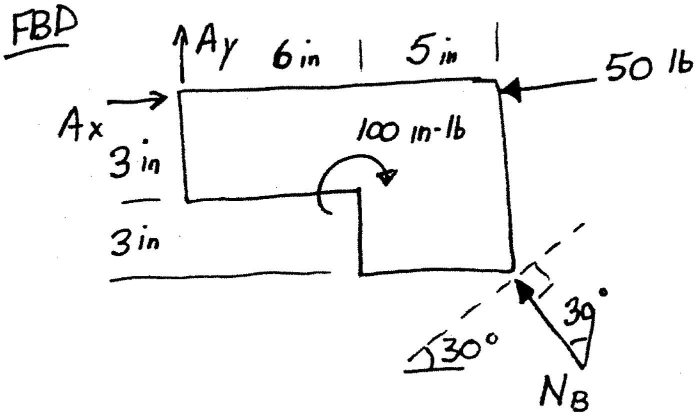
The plate shown is supported by a pin at A and a roller at B. The horizontal applied for F has a magnitude of 50 lb and the incline angle θ of the surface supporting the roller is 30 degrees. Determine the reactions at each support. Express each reaction as a magnitude and direction.



Given: $F = 50$ lb, $\theta = 30°$, pin at A, roller at B

Find: R_A, R_B mag + dir

FBD



PLAN
$\Sigma M_A : N_B$
$\Sigma F_x : N_B, A_x$
$\Sigma F_y : N_B, A_y$
Check with ΣM_B

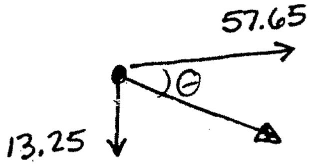
$$\overset{+}{\circlearrowleft} \Sigma M_A = N_B \cos 30 \overset{\text{in}}{(11)} - N_B \sin 30 \overset{\text{in.}}{(6)} - \overset{\text{in-lb}}{100} = 0$$

$$6.53\, N_B = 100 \qquad N_B = 15.3 \text{ lb}$$

$$\rightarrow \Sigma F_x = 0 = A_x - 50 - 15.3 \sin 30$$

$$A_x = 57.65 \text{ lb}$$

$$\uparrow \Sigma F_y = 0 = A_y + 15.3 \cos 30 \qquad A_y = -13.25 \text{ lb}$$



$$|A| = \sqrt{57.65^2 + 13.25^2}$$

$$= 59.2 \text{ lb.}$$

$$\tan\theta = \frac{13.25}{57.65} \qquad \theta = 12.9°$$

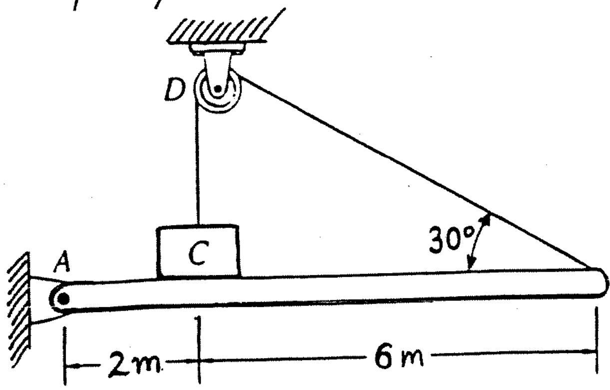
$N_B = 15.3$ lb ↖ 30°

$A = 59.2$ lb ↘ 12.9°

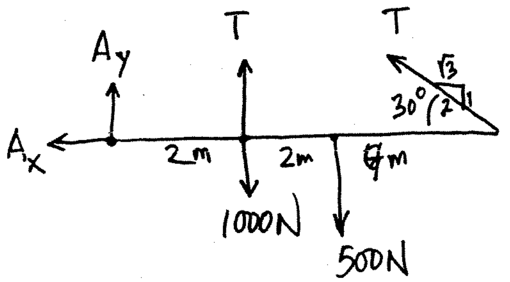
The 1000 N block C rests on a 500 N bar AB. The cable connecting C and B passes over a frictionless pulley at D. Determine the reactions at A and the tension in the cable.

Given: $W_c = 1000$ N
$W_{bar} = 500$ N
pulley is frictionless

FIND: reactions at A
T_{BC}



FBD



Plan

$$\Sigma M_A = 0 \Rightarrow T$$
$$\Sigma F_x = 0 \Rightarrow A_x, T$$
$$\Sigma F_y = 0 \Rightarrow A_y, T$$

$$\overset{+}{\circlearrowleft} \Sigma M_A = 0 = 2T + \left(\tfrac{1}{2}T\right)(8) - 1000(2) - 500($$

$$6T = 4000$$

$$\boxed{T = 666.67 \text{ lb}}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0 = -A_x - \frac{\sqrt{3}}{2}T$$

$$A_x = -\frac{\sqrt{3}}{2}\left(\overset{666.67}{\cancel{333.3}} \text{ lb}\right)$$

$$= -\overset{577.34}{\cancel{288.67}} \text{ lb}$$

$$\boxed{A_x = 577.34 \text{ lb} \rightarrow}$$

$$+\uparrow \Sigma F_y = 0 = A_y + T + \tfrac{1}{2}T - 1000 -$$

$$A_y = \overset{1500}{\cancel{1000}} - \frac{3}{2}T$$

$$= \overset{1500}{\cancel{1000}} - \frac{3}{2}\left(\overset{666.67}{\cancel{333.33}}\right)$$

$$\boxed{A_y = 500 \text{ lb.} \uparrow}$$

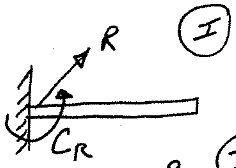
For the cantilever beam OA, with the applied moment and forces as shown, determine:

a.) the resultant of the force-couple system at the wall (point O)

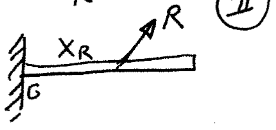
b.) the magnitude, direction, and location of the simplest resultant applied on the beam OA

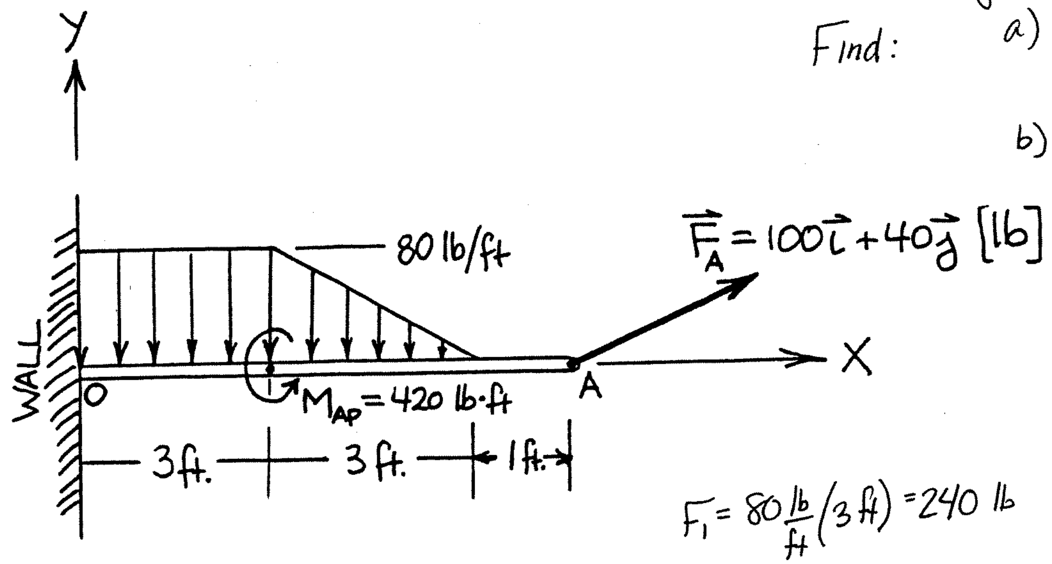
Given: figure

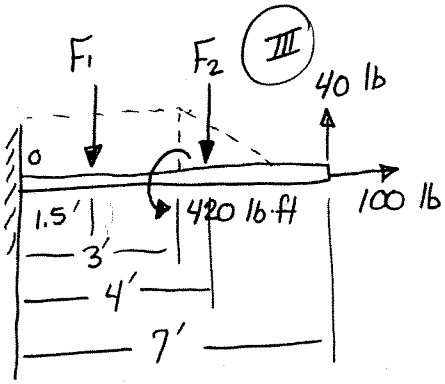
Find: a)



b)







$$F_1 = 80\frac{lb}{ft}(3\,ft) = 240\ lb$$

$$F_2 = \frac{1}{2}\left(80\frac{lb}{ft}\right)(3\,ft) = 120\ lb$$

a)

$$\bar{R} = 100\hat{i} - 320\hat{j}\ lb$$

$$C_R = -140\ lb\cdot ft$$

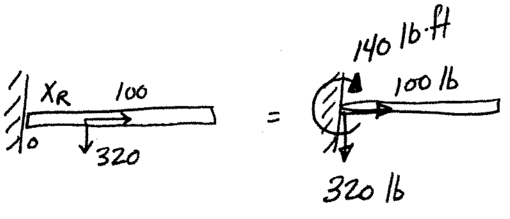
$$\bar{R} = \Sigma\bar{F} \quad \uparrow R_y = 40 - 240 - 120 = -320$$

$$R_x = 100$$

$$\Sigma M_o^{I} = \Sigma M_o^{III}$$

$$C_R = 40(7) - 120(4) - 240(1.5) + 420 \quad lb\cdot ft$$

$$\Sigma M_o^{II} = \Sigma M_o^{III}$$



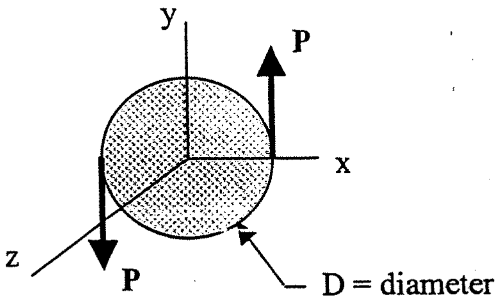
$$320(X_R) = 140 \qquad X_R = 0.438\ ft$$

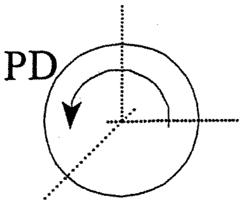
b)

$$\bar{R} = 100\hat{i} - 320\hat{j}\ lb$$

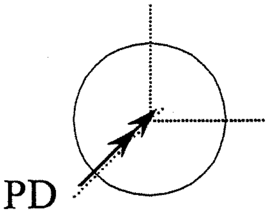
$$X_R = 0.438\ ft$$

Which of the following figures represents a system that is equivalent to the couple shown at the right?

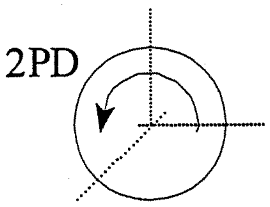




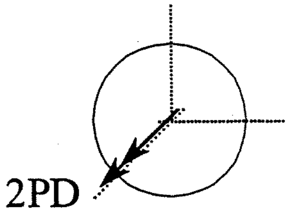
1



2



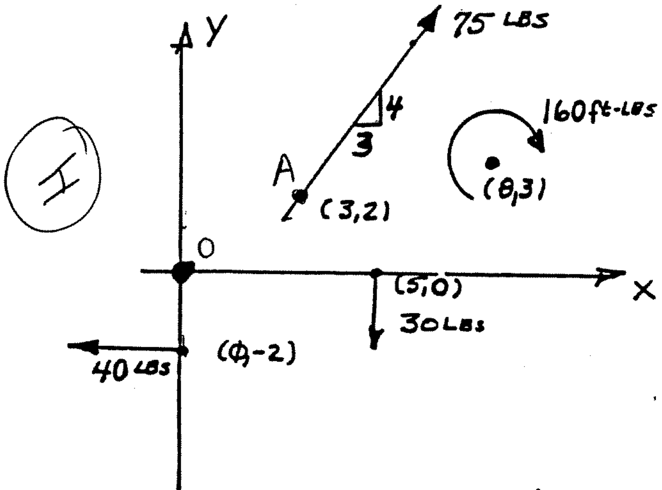
3



4

- [✓ A) 1 only]
- B) 1 and 2
- C) 3 and 4
- D) 2 only

Determine THE SIMPLEST RESULTANT of the three forces and one couple, system shown. Determine the location where this resultant crosses the "X" axis.



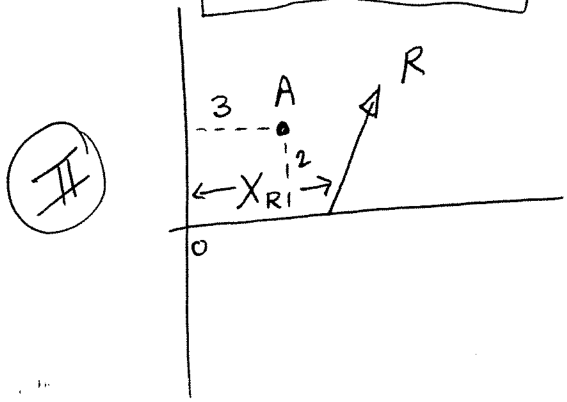
$\vec{R}$ = $5\hat{i} + 30\hat{j}$ lb

"$\bar{X}$" = -10 ft.

Find: R, X_R

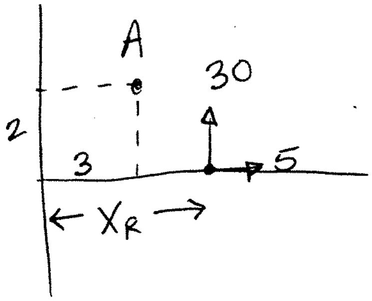
$$\bar{R} = \Sigma F = 75\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right) - 40\hat{i} - 30\hat{j} \text{ lb}$$

$$\bar{R} = 5\hat{i} + 30\hat{j} \text{ lb}$$



$$\overset{+}{\circlearrowleft} \quad \Sigma M_A^{I} = \Sigma M_A^{II}$$

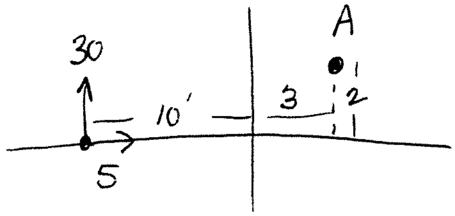
$$-30\,\text{lb}(2) - 40(4) - 160 = 30(X_R - 3) + 5(2)$$



$$-380 = 30X_R - 80$$

$$-300 = 30X_R \qquad X_R = -10$$

Check



$\Sigma M_A = 5(2) - 30(13) = -380$ OK

You have just been hired by Boeing to analyze their new arm rest design. The triangular arm rest is pin connected to the base of the seat (point A) and rests on the smooth surface of the seat back (point C). The passenger will likely lean on the horizontal top surface (CB) with the distributed load shown. Under these conditions, what will be the magnitude of the force exerted by the pin on the armrest at A? (Neglect the weight of the armrest.)

Read FIND $|A|$

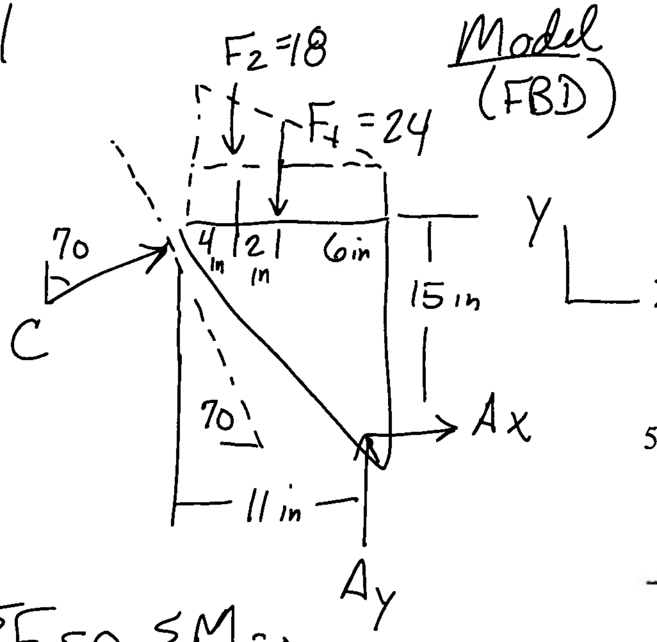
$F_1 = (2\ \text{lb/in})(12\ \text{in})$

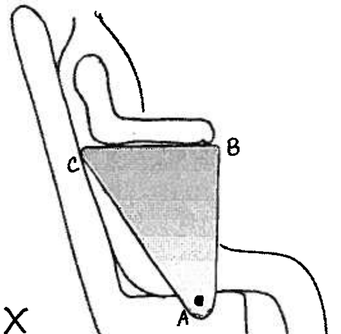
$= 24\ \text{lb}$

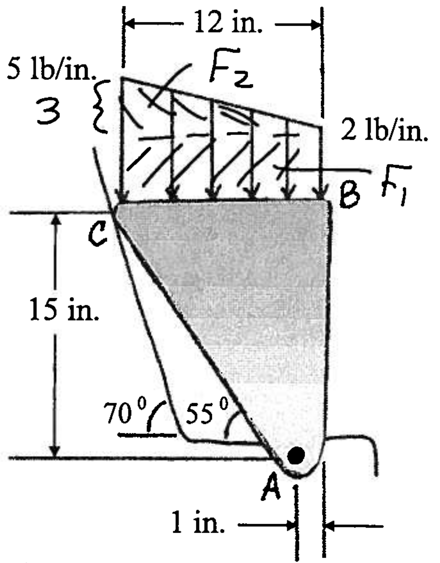
$F_2 = \frac{1}{2}(3\ \text{lb/in})(12\ \text{in})$

$= 18\ \text{lb}$

Model (FBD)







Apply $\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M = 0$

Solve $\xrightarrow{+} \Sigma F_x = 0 = A_x + C \sin 70$

$\uparrow \Sigma F_y = 0 = A_y - 18 - 24 + C \cos 70$

$\circlearrowleft{+}\ \Sigma M_A = 0 = 24(5) + 18(7) - C \sin 70 (15) - C \cos 70 (11)$

$246 = 17.86\, C \qquad C = 13.77\ \text{lb}$

$A_x = -13.77(\sin 70) = -12.94$

$A_y = 18 + 24 - 13.77 \cos 70 = 37.29$

$|A| = \sqrt{12.94^2 + 37.29^2} = 39.47$

$\boxed{|A| = 39.5\ \text{lb}}$

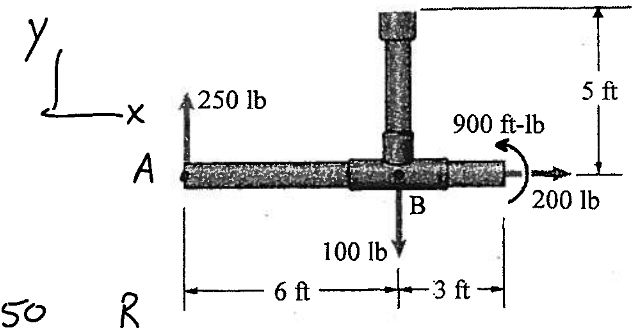
11:18

Check $\Sigma M_C \overset{?}{=} 0 = -18(4) - 24(6) + 37.29(11)$ ✓

$+ (-12.94)(15) = 0.09$

The rigid pipes shown are subjected to three forces and a couple given in the figure.

a) Replace the loads by an equivalent force and couple acting at point B. Report the magnitude and direction of both the force and the couple in the box and sketch these on the blank figure below.



Read FIND R, C_R at B

Apply $\Sigma F = R$

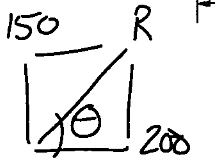
$\Sigma M_B = C_R$

Solve $\Sigma F = 250\hat{j} - 100\hat{j} + 200\hat{i}$

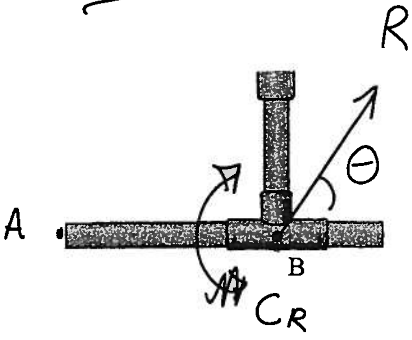
$R = 200\hat{i} + 150\hat{j}$ lb

$|R| = \sqrt{200^2 + 150^2} = 250$ lb

$\tan\theta = 150/200 \quad \theta = 36.9^\circ$



Model



$\circlearrowleft +$ $\Sigma M_B = 900 - 250(6) = -600$ lb·ft

$= 600$ lb ft $\circlearrowright$

check $\Sigma M_A = -100(6) + 900 = 150(6) - 600$

$300 = 300$ ✓

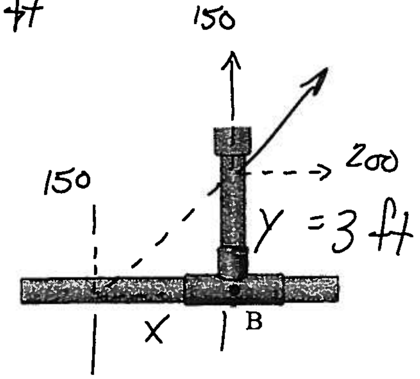
$R = 250$ lb
$\theta = 36.9°$ $\circlearrowright$
$C_R = 600$ lb·ft CW

b) Now replace the loads by an equivalent single resultant force. Sketch this resultant and label it's location on the figure below.

$\Sigma M_B = -600 \text{ lb·ft} = -200(y) \quad y = 3 \text{ ft}$

OR

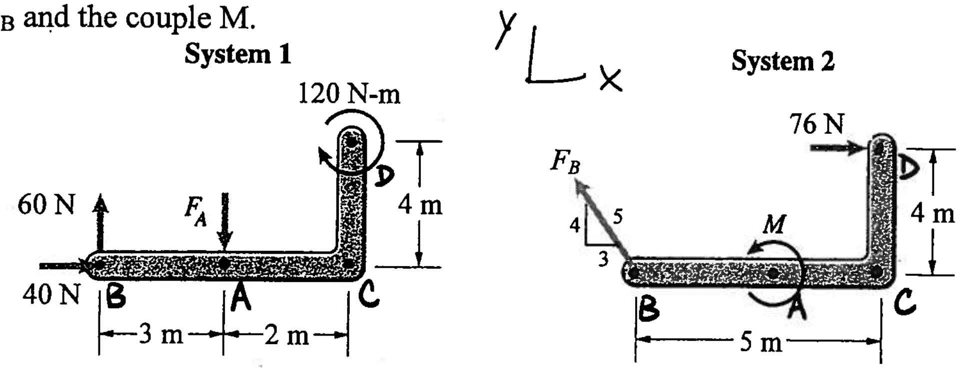
$-600 \text{ lb·ft} = -150(x) \quad x = 4 \text{ ft}$



Two equivalent systems of forces and moments act on the L-shaped bar. Determine the magnitudes of forces F_A and F_B and the couple M.

Read FIND:

F_A, F_B, M



Apply $\Sigma F^1 = \Sigma F^2$

$\Sigma M^1 = \Sigma M^2$

← Model ↗

Solve → $\Sigma F_x{}^1 = 40$ $\Sigma F_x{}^2 = 76 - \frac{3}{5} F_B$

$40 = 76 - \frac{3}{5} F_B$ $\frac{3}{5} F_B = 76 - 40$ $F_B = 60$ N

↑ $\Sigma F_y{}^1 = 60 - F_A$ $\Sigma F_y{}^2 = \frac{4}{5}(60)$

$60 - F_A = 48$ $60 - 48 = F_A$ $F_A = 12$ N

↺ $\Sigma M_B^1 = -120 - 12(3)$ $\Sigma M_B^2 = M - 76(4)$

$-120 - 36 = M - 304$ $M = 148$ N·m

Check

$\Sigma M_C^1 = -120 + 12(2) - 60(5) = -396$ N·m

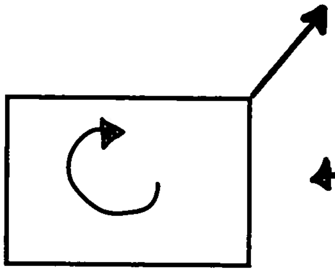
$\Sigma M_C^2 = 148 - 76(4) - \frac{4}{5}(60)(5) = -396$ ✓

$F_A = 12$ N
$F_B = 60$ N
$M = 148$ N·m

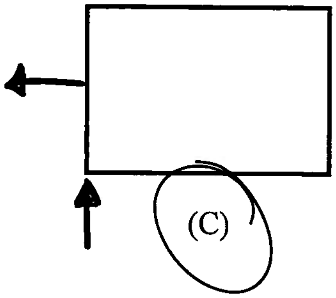
4.1 (3%) Select the letter of the one system below that is definitely NOT equivalent to the others.

All three systems could be equivalent

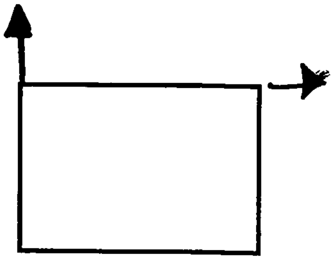
(A)



(B)



[✓ (C)]



(D)

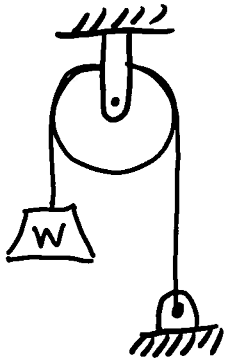
4.2 (3%) Assuming the pulley is frictionless and the system is in static equilibrium as shown, what is the tension in the rope?

A) Can't be determined

B) 2W

[✓ C) W]

D) W/2



4.3 (4%) Assuming the platform below the Players sign shown in the photo and simplified 2-D model below is attached to the building at B with a rigid fixed connection, draw a complete and accurate FBD of the platform AB in the box provided, labeling the weight as W.