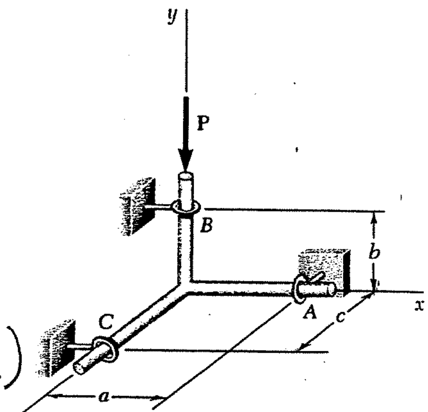
Three rods are welded together to form a 'corner', which is supported by three eyebolts (slider bearings). Neglecting friction, determine the reactions at A, B and C when P = 240 N, a = 120 mm, b = 80 mm, and c = 100 mm.

Given: $P = 240\ N$, $a = 120\ mm$, $b = 80\ mm$, $c = 100\ mm$

Find: Reactions at A, B, C



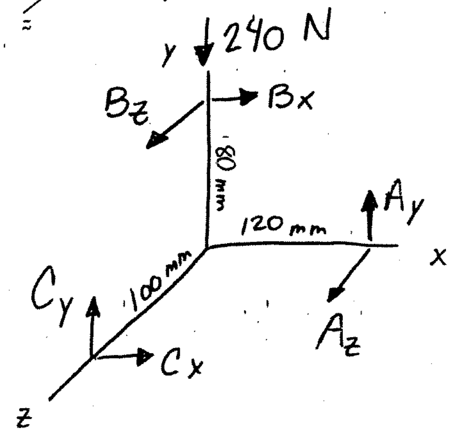
PLAN (one of several possible plans)

(2) $\Sigma \bar{M}_B : A_y, A_z, C_x, C_y$ (3 eq. 4 unk.)

(1) $\Sigma F_y : A_y, C_y$ (Because B does not have a y comp. this equation does not add any unk.)

(3) $\Sigma F_z : A_z, B_z$

(4) $\Sigma F_x : B_x, C_x$



$$\Sigma F_y = 0 = C_y + A_y - 240 \qquad C_y = 240 - A_y$$

$$\Sigma \bar{M}_B = (120\hat{i} - 80\hat{j}) \times (A_y\hat{j} + A_z\hat{k}) + (-80\hat{j} + 100\hat{k}) \times (C_x\hat{i} + C_y\hat{j}) = 0$$

$$= 120 A_y \hat{k} - 120 A_z \hat{j} - 80 A_z \hat{i} - (-80 C_x)\hat{k} + 100 C_x \hat{j} - 100 C_y \hat{i} = 0$$

$\hat{i}$: $-80 A_z - 100 C_y = 0$ $\qquad A_z = -1.25 C_y$

$\hat{j}$: $-120 A_z + 100 C_x = 0$ $\qquad 100 C_x = 120(-1.25 C_y)$ $\qquad C_x = -1.5 C_y$

$\hat{k}$: $120 A_y + 80 C_x = 0$ $\qquad 120 A_y = -80(-1.5 C_y) = 120 C_y \Rightarrow A_y = C_y$

using ΣF_y above $C_y = 240 - C_y$

$$\boxed{\begin{aligned} C_y &= 120\ N \\ C_x &= -180\ N \\ A_z &= -150\ N \\ A_y &= 120\ N \end{aligned}}$$

$\Sigma F_z = 0 = B_z + A_z = 0$

$\Sigma F_x = 0 = B_x + C_x = 0$

$$\boxed{\begin{aligned} B_z &= +150\ N \\ B_x &= +180\ N \end{aligned}}$$

Alternate Solution

ΣM_C : B_x, B_z, A_y, A_z , ΣF_z : A_z, B_z

$$\Sigma M_C = (120\hat{i} - 100k) \times (A_y\hat{j} + A_z\hat{k}) + (80\hat{j} - 100\hat{k}) \times (B_x\hat{i} + B_z\hat{k} - 240\hat{j})$$

$$120 A_y \hat{k} - 120 A_z \hat{j} + 100 A_y \hat{i} - 80 B_x \hat{k} + 80 B_z \hat{i} - 100 B_x \hat{j} - 24000\,\tilde{i}$$

i: $100 A_y + 80 B_z - 24000 = 0$

j: $-120 A_z - 100 B_x = 0$

k: $120 A_y - 80 B_x = 0$

$\Sigma F_z = 0$

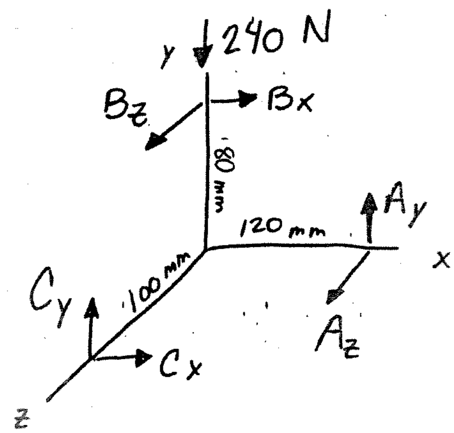
$A_z + B_z = 0$

$A_z = -B_z$

$B_x = 1.5 A_y$

$-120 A_z = 100(1.5 A_y)$

$A_y = -0.8 A_z = +0.8 B_z$



$100(0.8 B_z) + 80 B_z = 24000$

$160 B_z = 24000$

$B_z = +150$

$A_y = +120$

$A_z = -150$

$B_x = 180$

$\Sigma F_x = 0 = C_x + B_x = 0$ $\quad C_x = -B_x \quad C_x = -180$

$\Sigma F_y = 0 = A_y + C_y - 240 \quad C_y = 240 - 120 = 120$

$C_y = 120$

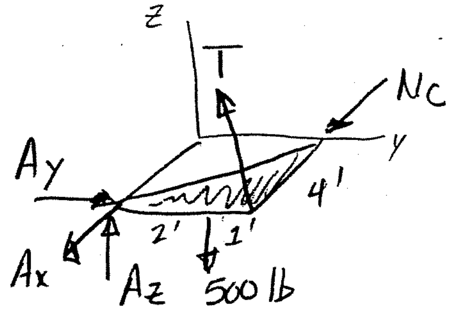
The horizontal rigid plate **ABC** is loaded with a weight **W** = 500 lb and is held in place by a ball-in-socket at the wall at **A**, a rope from **B** to **D** and by a frictionless point contact at the other wall (**C**: yz – plane). Neglect the weight of the plate itself.

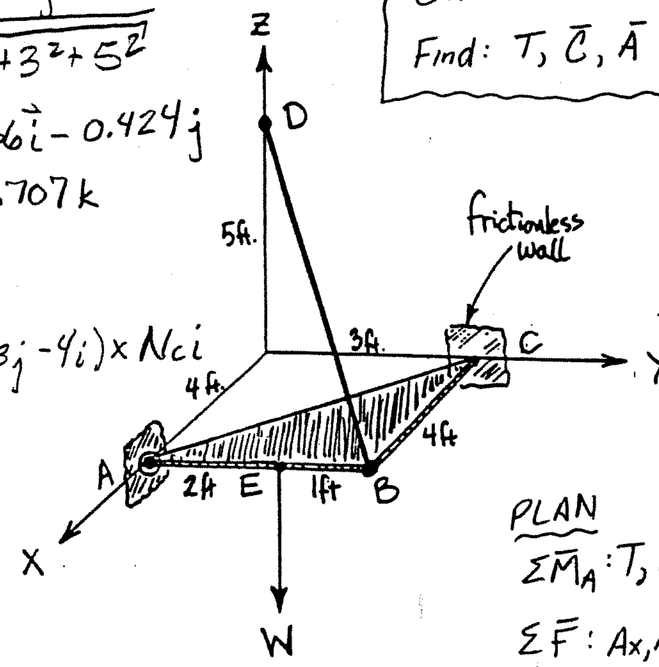
Determine the tension in the rope $\mathbf{T_{BD}}$, the support reaction at **C**, and the total force supported at A.

Given: W=500 lb
Find: $T, \bar{C}, \bar{A}$

$$\lambda_{BD} = \frac{-4i - 3j + 5k}{\sqrt{4^2+3^2+5^2}}$$

$$\lambda_{BD} = -0.566\vec{i} - 0.424\vec{j} + 0.707k$$





PLAN
$\Sigma \bar{M}_A : T, N_c$
$\Sigma F : A_x, A_y, A_z$
T, N_c

k
i ⊕ j

$$\Sigma \vec{M}_A = 0 = 2j \times (-500k) + 3j \times T(\lambda_{BD}) + (3j - 4i) \times N_c i$$

$$= -1000\vec{i} - 3N_c k + 1.698 T\vec{k} + 2.121 T\vec{i}$$

$i: \; -1000 + 2.121T = 0 \qquad T = 471.5$

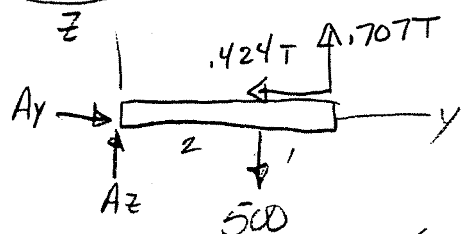
$k: \; -3N_c + 1.698(471.5) = 0 \qquad N_c = 267$

$$\Sigma \vec{F} = 0 = A_x i + A_y j + A_z k - 500k + 267i + 471.5\,\lambda_{BD} = 0$$

$i: \; A_x + 267 - 471.5(.566) \qquad A_x = 0$

$j: \; A_y - 471.5(.424) \qquad A_y = 200$

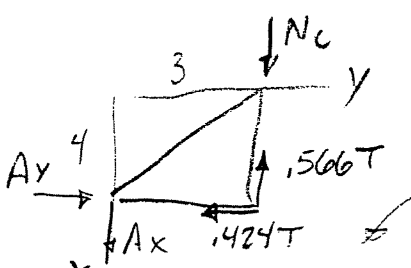
$k: \; A_z - 500 + 471.5(.707) \qquad A_z = 167$



$\Sigma F_y = 0 \quad A_y = .424(471.5) = 200$

$\Sigma F_z = 0 \quad A_z = 500 - .707(471.5) \quad A_z = 167$

$\Sigma M_A = 0 \quad 500(2) = 3(.707T) \quad T = 471.5$



$\Sigma F_x = 0 \quad A_x = 0$

because

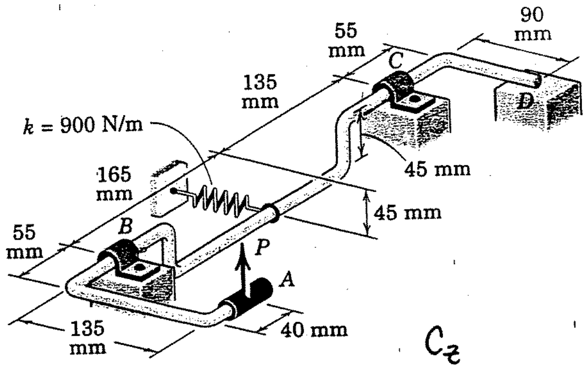
$\Sigma M_A = 0 \quad .566(471.5) = N_c = 267$

$|T_{BD}| =$ 471.5 lb

$\vec{C} =$ $267\,\vec{i}$

$\vec{A} =$ $200j + 167\vec{k}$

The spring of modulus k = 900 N/m is stretched a distance d = 60 mm when the mechanism is in the position shown. Calculate the force P_{min} required to initiate rotation about the hinge axis BC, and determine the corresponding magnitudes of the bearing forces, which are perpendicular to BC. What is the normal reaction force at D if $P = P_{min} / 2$?



Given: $k_{spring} = 900$ N/m

$\Delta_{spring} = 0.06$ m

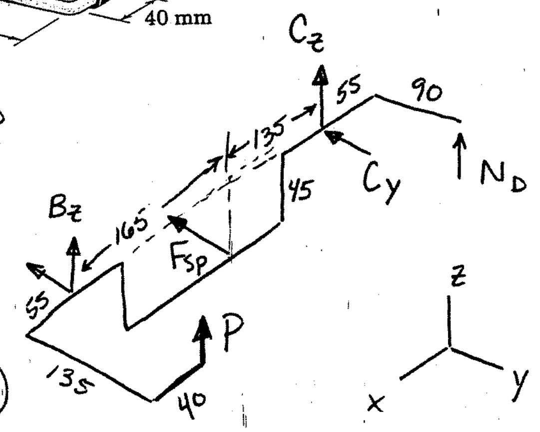
Find: a) P_{min} to start rotation, $|B|$, $|C|$

b) N_D when $P = P_{min}/2$

$F_{sp} = k\Delta = 900 \text{ N/m} (0.06\text{m}) = 54$ N

a) When rotation starts, $N_D = 0$

FBD



PLAN $\Sigma\bar{M}_B : C_y, C_z, P$, $\Sigma\bar{F} : B_y, B_z, C_y, C_z, P$
(6 equations, 5 unknowns)

$$\Sigma\bar{M}_B = (-165\hat{\imath} - 45\hat{k}) \times (-54\hat{\jmath}) + (-300\hat{\imath}) \times (-C_y\hat{\jmath} + C_z\hat{k}) + (15\hat{\imath} + 135\hat{\jmath}) \times P\hat{k} = 0$$

$$= 8910\hat{k} - 2430\hat{\imath} + 300 C_y \hat{k} - (-300 C_z)\hat{\jmath} - 15P\hat{\jmath} + 135P\hat{\imath} = 0$$

$i: (-2430 + 135P) = 0$

$j: (300 C_z - 15(18)) = 0$

$k: (8910 + 300 C_y) = 0$

$$\boxed{P = 18 \text{ N} \quad C_z = 0.9 \text{ N} \quad C_y = -29.7 \text{ N}}$$

$$\Sigma\bar{F} = 18\hat{k} + 0.9\hat{k} + 29.7\hat{\jmath} - 54\hat{\jmath} + B_z\hat{k} - B_y\hat{\jmath} = 0$$

$i: 0$

$j: 29.7 - 54 - B_y = 0$

$k: 18 + 0.9 + B_z = 0$

$$\boxed{B_y = -24.3 \text{ N} \quad B_z = -18.9 \text{ N}}$$

b) $P = 9$ $\quad \Sigma M_{BC} = 9(135) - 54(45) + N_D(90) = 0$

$$\boxed{N_D = 13.5 \text{ N}}$$

Checks

$\Sigma M_{BC} = 0$

$P(135) - 54(45) = 0$

$P = 18$ ok

$(\Sigma M_B)_y = C_z(300) - P(15) =$

$C_z = 0.9$ N ok

$(\Sigma M_B)_z = C_y(300) + 54(165)$

$C_y = 29.7$ ok

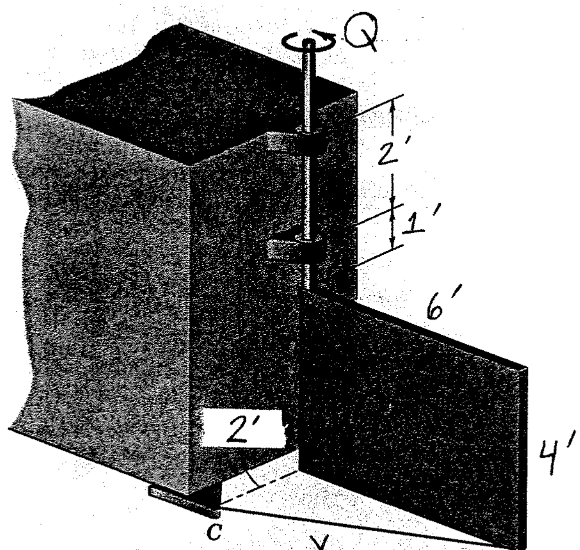
$(\Sigma M_C)_y = B_z(300) + 18(315)$

$B_z = -18.9$ ok

$(\Sigma M_C)_z = 54(135) + B_y(300$

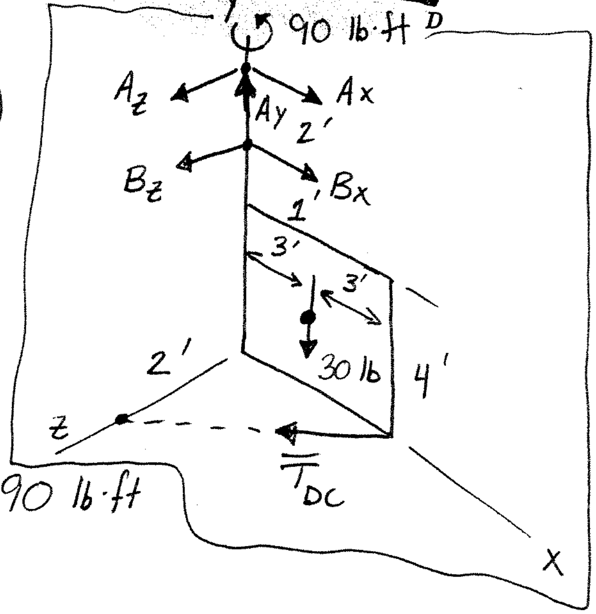
$B_y = -24.3$ ok

The uniform 30 lb plate is welded to the vertical shaft, which is supported by a thrust bearing at A and a journal bearing at B. Calculate the magnitude of the force supported by bearing B during application of the 90 ft-lb couple, Q, to the shaft. The cable from C to D maintains static equilibrium by preventing the plate and shaft from turning. The weight of the assembly is carried entirely by the thrust bearing at A.



Given: $Q = 90$ ft·lb, $W = 30$ lb

FIND: $|B|$



$$\bar{T}_{DC} = T_{DC}\left(\frac{-6\hat{i}+2\hat{k}}{\sqrt{40}}\right) = T_{DC}\left(-\frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{k}\right)$$

$$\Sigma \bar{M}_A = 0 = -2\hat{j} \times (B_x\hat{i} + B_z\hat{k})$$

$$(+3\hat{i} - 5\hat{j}) \times (-30\hat{j})$$

$$+(6\hat{i} - 7\hat{j}) \times \left(\frac{-3}{\sqrt{10}}T_{DC}\hat{i} + \frac{1}{\sqrt{10}}T_{DC}\hat{k}\right) + 90 \text{ lb·ft}$$

i: $-2B_z - \frac{7}{\sqrt{10}}T_{DC} = 0$ $B_z = 52.50$

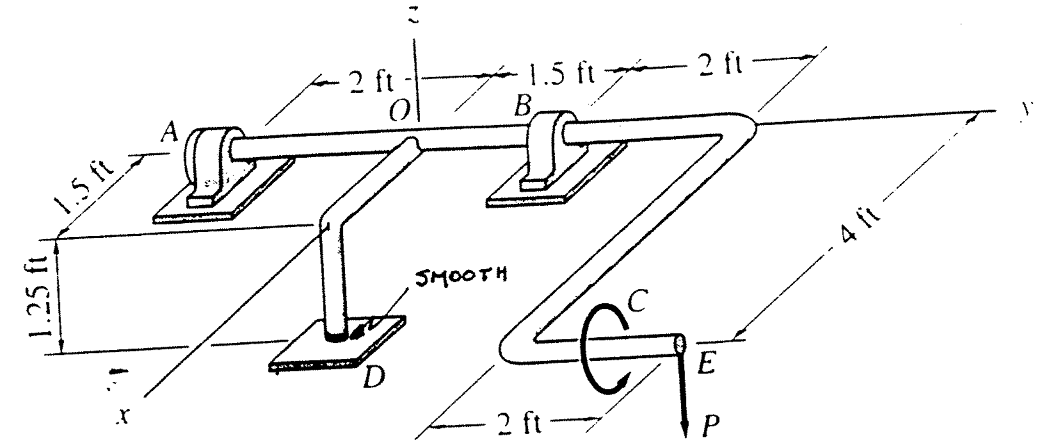
j: $-\frac{6}{\sqrt{10}}T_{DC} + 90 = 0 \rightarrow T_{DC} = 47.43$ lb.

k: $2B_x - 90 - \frac{21}{\sqrt{10}}T_{DC} = 0$ $B_x = 202.5$

$$|B| = \sqrt{52.50^2 + 202.5^2} = 209.2 \text{ lb.}$$

$$\boxed{|B| = 209.2 \text{ lb}}$$

#2. The crank is supported by a thrust bearing at A, a slider bearing at B and a smooth surface at D. Calculate the reactions at A, B, and D if the load **P** = 210 lbs and the applied couple C = 840 ft lbs.

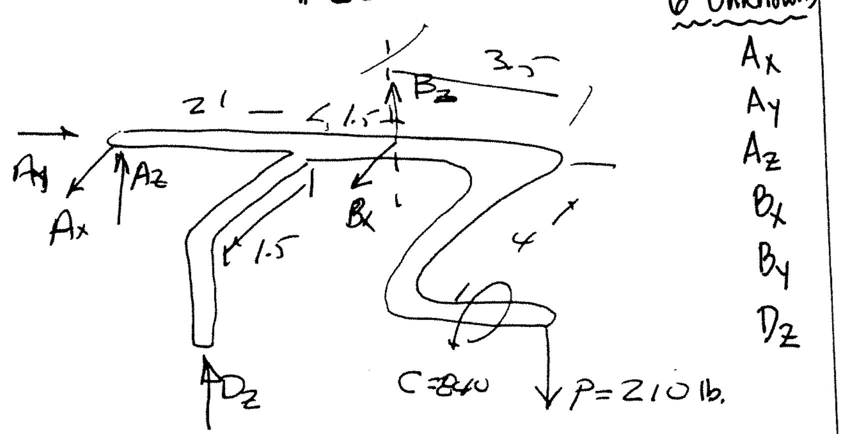


$\vec{A} = -720\,\hat{k}$

$\vec{B} = -190\,\hat{k}$

$\vec{D} = 1120\,\hat{k}$

FBD



6 Unknowns

A_x, A_y, A_z, B_x, B_y, D_z

① $\Sigma F_y = 0 = A_y = 0$

② $\Sigma M_{y\,axis} = 840 + (4)(210) - 1.5 D_z = 0$

$\therefore \boxed{D_z = \frac{840+840}{1.5} = \frac{1680}{1.5} = 112}$

③ $\Sigma M_{z\,axis}$ at B $= 3.5 A_x = 0$

$\therefore \boxed{A_x = 0}$

④ $\Sigma F_x = A_x + B_x = 0 = 0 + B_x$

$\therefore \boxed{B_x = 0}$

⑤ AT "A" $\Sigma M_{x\,axis} = 2.0 D_z + 3.5 B_z - (7.5)(210) = 0$

$\therefore (2)(1{,}120) + 3.5 B_z - 1575 = 0 \quad \therefore 3.5 B_z = 1575 - 2240 =$

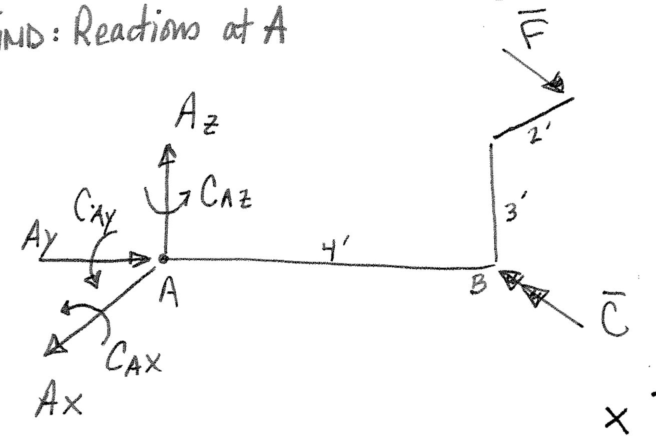
$\boxed{B_z = \frac{-665}{3.5} = -190 \text{ lb}}$

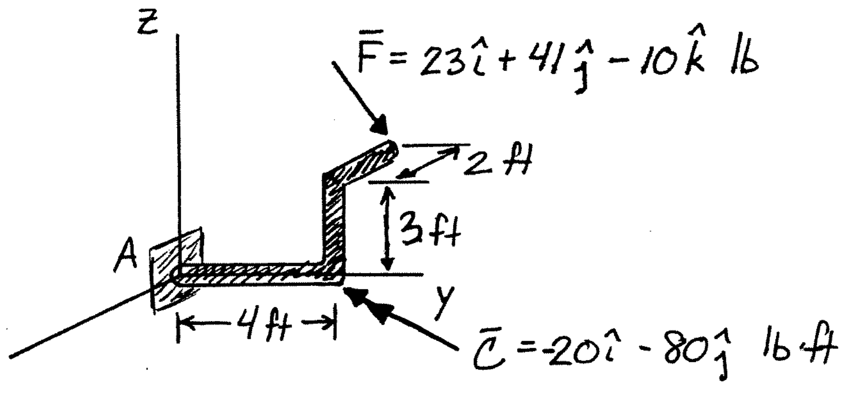
⑥ $\Sigma F_z = A_z + D_z + B_z - 210 = 0$

$\therefore A_z = 210 - D_z - B_z = 210 - 1{,}120 - (-190) = \boxed{-720 \text{ lb} = A_z}$

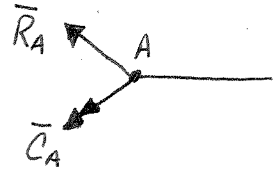
The bent bar is rigidly fixed to the wall at A and supports a force **F** and a couple **C** as shown. Determine the reactions at A required to maintain equilibrium.

FIND: Reactions at A





You could also model the fixed connection as



$\Sigma \bar{F}_A = 0 = \bar{F} + \bar{R}_A$

$23\hat{i} + 41\hat{j} - 10\hat{k} + A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = 0$

$i: \; 23 + A_x = 0 \qquad A_x = -23 \text{ lb}$

$j: \; 41 + A_y = 0 \qquad A_y = -41 \text{ lb}$

$k: \; -10 + A_z = 0 \qquad A_z = 10 \text{ lb.}$

$\Sigma \bar{M}_A = 0 = \bar{C}_A + \bar{C} + (-2\hat{i} + 4\hat{j} + 3\hat{k}) \times \bar{F} = 0$

$$C_{Ax}\hat{i} + C_{Ay}\hat{j} + C_{Az}\hat{k} - 20\hat{i} - 80\hat{j} + \begin{vmatrix} i & j & k \\ -2 & 4 & 3 \\ 23 & 41 & -10 \end{vmatrix} = 0$$

$i: \; C_{Ax} - 20 + [(4)(-10) - (3)(41)] = 0 \qquad C_{Ax} = 183 \text{ lb·ft}$

$j: \; C_{Ay} - 80 - [(-2)(-10) - (3)(23)] = 0 \qquad C_{Ay} = 31 \text{ lb·ft}$

$k: \; C_{Az} + [(-2)(41) - (4)(23)] = 0 \qquad C_{Az} = 174 \text{ lb·ft}$

$$\boxed{\begin{aligned} \bar{R}_A &= -23\hat{i} - 41\hat{j} + 10\hat{k} \text{ lb} \\ \bar{C}_A &= 183\hat{i} + 31\hat{j} + 174\hat{k} \text{ lb·ft} \end{aligned}}$$

Check does ΣM_B really equal zero?

$-20\hat{i} - 80\hat{j} + (-2\hat{i} + 3\hat{k}) \times (23\hat{i} + 41\hat{j} - 10\hat{k})$

$+ (-4\hat{j}) \times (-23\hat{i} - 41\hat{j} + 10\hat{k})$

$+ 183\hat{i} + 31\hat{j} + 174\hat{k} = 0 \,?$

$i: -20 - (3)(41) + (-4)(10) + 183 \overset{?}{=} 0$ yes $\qquad j: -80 - 20 + 69 + 31 \overset{?}{=} 0$ yes $\qquad k:$ yes