Name Solution

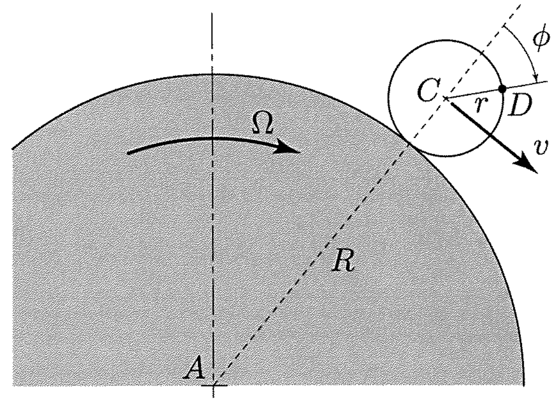
THE PENNSYLVANIA STATE UNIVERSITY
Department of Engineering Science and Mechanics

Engineering Mechanics 112H — Exam 3
April 24, 2006

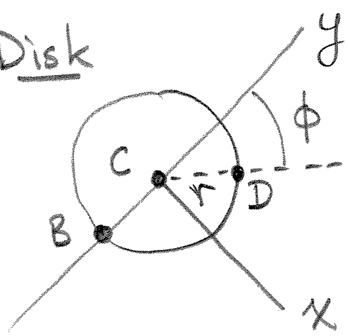
Problem 1	*Problem 2*	*Problem 3*	*Problem 4*	**Total**

Problem 1 (31 pts)

The disk of radius r rolls without slipping over the exterior of a large drum of radius R. The drum rotates clockwise at constant angular speed Ω. In the position shown, the center of the disk has speed v, which is increasing at the rate $\dot{v}$. Derive expressions for the velocity and acceleration of point D, which is situated at an arbitrary angle ϕ relative to the radial line connecting the centers.



The Disk



We are given that: $\vec{V}_c = v\,\hat{i}$

In addition, since C is moving in a circle centered at A

$$\vec{a}_c = \dot{v}\,\hat{i} - \frac{v^2}{R+r}\,\hat{j}$$

Since C rolls without slip on the drum, we also know $\vec{V}_B = R\Omega\,\hat{i}$. Now:

$$\vec{V}_c = \vec{V}_B + \vec{\omega}_c \times \vec{r}_{c/B} \Rightarrow v\,\hat{i} = R\Omega\,\hat{i} + \omega_c\,\hat{k} \times r\,\hat{j}$$

$$\Rightarrow v = R\Omega - r\omega_c \Rightarrow \omega_c = \frac{R\Omega - v}{r}$$

Since ω_c is valid for all time:

$$\alpha_c = \dot{\omega}_c = \frac{1}{r}\left(R\dot{\Omega} - \dot{v}\right)$$

since $\Omega = \text{const} \Rightarrow \alpha_c = -\frac{\dot{v}}{r}$

Now: $\vec{V}_D = \vec{V}_c + \vec{\omega}_c \times \vec{r}_{D/c}$

$$\vec{V}_D = v\hat{i} + \frac{1}{r}(R\Omega - v)\hat{k} \times r(\sin\phi\,\hat{i} + \cos\phi\,\hat{j})$$

$$= \left[v - (R\Omega - v)\cos\phi\right]\hat{i} + (R\Omega - v)\sin\phi\,\hat{j}$$

$$\boxed{\vec{V}_D = \left[v(1+\cos\phi) - R\Omega\cos\phi\right]\hat{i} + (R\Omega - v)\sin\phi\,\hat{j}}$$

$$\vec{a}_D = \vec{a}_c + \vec{\alpha}_c \times \vec{r}_{D/c} - \omega_c^2\,\vec{r}_{D/c}$$

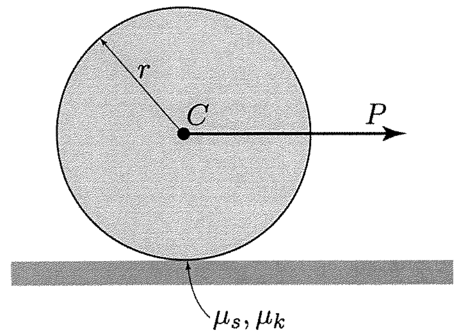
$$= \dot{v}\hat{i} - \frac{v^2}{R+r}\hat{j} - \frac{\dot{v}}{r}\hat{k} \times r(\sin\phi\,\hat{i} + \cos\phi\,\hat{j})$$

$$- \left(\frac{R\Omega - v}{r}\right)^2 r(\sin\phi\,\hat{i} + \cos\phi\,\hat{j})$$

$$\boxed{\begin{aligned}\vec{a}_D = &\left[\dot{v} + \dot{v}\cos\phi - \frac{(R\Omega - v)^2}{r}\sin\phi\right]\hat{i} \\ &+ \left[-\frac{v^2}{R+r} - \dot{v}\sin\phi - \frac{(R\Omega - v)^2}{r}\cos\phi\right]\hat{j}\end{aligned}}$$

Problem 2 (31 pts)

The uniform disk of radius r and mass m is at rest when a force P begins pulling to the right on its center. Given that the coefficients of static and kinetic friction between the disk and the ground are μ_s and μ_k, respectively, and that P is sufficiently large to cause the disk to slip,

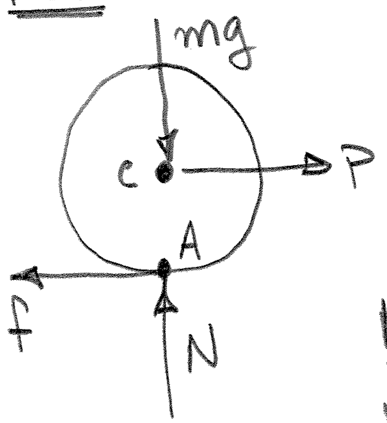


(a) determine the angular acceleration of the disk and the acceleration of its mass center C,

(b) state the condition for the admissibility of this solution, and

(c) determine the condition on P that satisfies the admissibility condition stated in (b).

FBD



Newton - Euler

$$\sum F_x: \; P - f = m a_{cx}$$

$$\sum F_y: \; N - mg = m a_{cy}$$

$$\sum M_c: \; -fr = I_c \alpha$$

Material: $f = \mu_k N \; ; \; I_c = \frac{1}{2} m r^2$

Kinematics: $\boxed{a_{cy} = 0}$

Solving: $N = mg$

$$P - \mu_k mg = m a_{cx} \Rightarrow \boxed{a_{cx} = \frac{P}{m} - \mu_k g}$$

$$-\mu_k \cancel{m} g \cancel{r} = \frac{1}{2} \cancel{m} r^{\cancel{2}} \alpha \Rightarrow \boxed{\alpha = -2 \frac{\mu_k g}{r}}$$

The admissibility condition concerns the motion at A. Since f is to the left, we must have that $a_{Ax} \geq 0$:

$$\vec{a}_A = \vec{a}_c + \vec{\alpha} \times \vec{r}_{A/c} \qquad (\omega = 0)$$

$$a_{Ax}\hat{i} + a_{Ay}\hat{j} = \left(\frac{P}{m} - \mu_k g\right)\hat{i} - 2\frac{\mu_k g}{r}\hat{k} \times (-r\hat{j})$$

$$\hat{i}: \quad a_{Ax} = \frac{P}{m} - \mu_k g - 2\mu_k g = \frac{P}{m} - 3\mu_k g$$

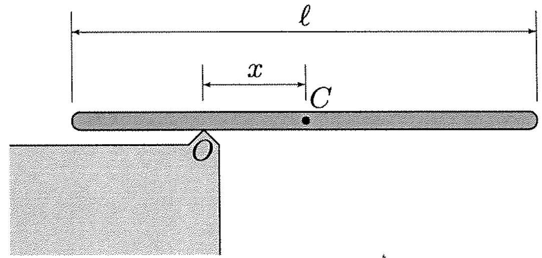
Since we want $a_{Ax} > 0$

$$\Longrightarrow \quad \boxed{P \geq 3\mu_k m g}$$

It is also true that $a_{cx} \geq 0$, but it is easily seen that this is also satisfied by the above condition.

Problem 3 (31 pts)

The uniform slender bar is released from rest in the horizontal position shown. Determine the value of x for which the angular acceleration is a maximum, and determine the corresponding angular acceleration α.



Need to find $\alpha(x)$ and then take $\frac{d\alpha}{dx} = 0$.

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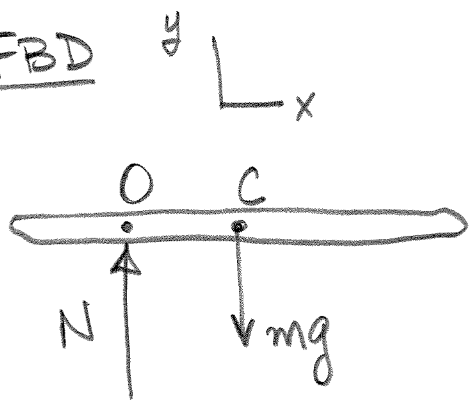


$\sum M_O : -mgx = I_O \alpha$

Material : $I_O = I_C + mx^2$

$I_O = \frac{1}{12} m\ell^2 + mx^2$

$= m\left(x^2 + \ell^2/12\right)$

Therefore: $\alpha = -g \dfrac{x}{x^2 + \ell^2/12}$

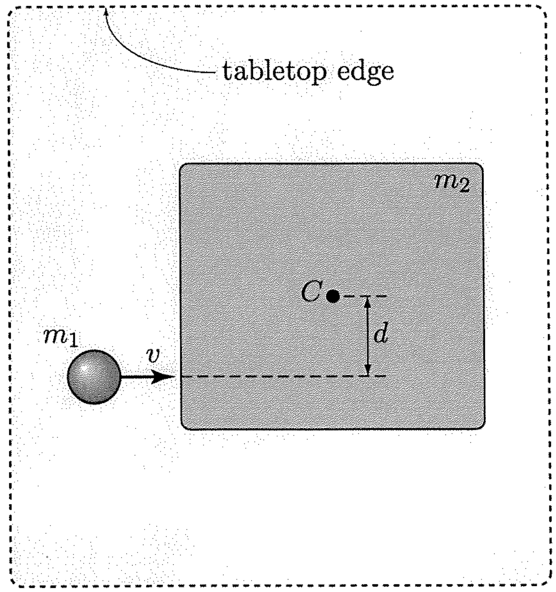
$$\frac{d\alpha}{dx} = -g\left[\frac{x^2 + \ell^2/12 - 2x^2}{\left(x^2 + \ell^2/12\right)^2}\right] = 0$$

$-x^2 = -\ell^2/12 \Rightarrow \boxed{x = \dfrac{\ell}{\sqrt{12}} = \dfrac{\ell}{2\sqrt{3}} = 0.289\,\ell}$

$$\boxed{\alpha_{MAX} = -g\,\frac{\ell/\sqrt{12}}{\ell^2/12 + \ell^2/12} = -\sqrt{3}\,\frac{g}{\ell}}$$

Problem 4 (7 pts)

The box of mass m_2 is at rest on a smooth horizontal tabletop when it is hit by a non-spinning ball of mass m_1 traveling at speed v, which then bounces off. The mass center of the box is at C and its mass is uniformly distributed. Describe the ensuing motion of the box. Be brief, but careful.



The impulsive force on the box due to m_1 is to the right and has no vertical component. In addition, it's line of action does not go through C. Therefore, the box will rotate ccw and C will move to the right.