Math 140 Spring 05 EXAMINATION I

1. (5 pts.) Determine the values of a for which $\lim_{x\to a} f(x)$ exists:

$$f(x) = \begin{cases} 1-x & \text{if } x < -5 \\ x & \text{if } -5 \le x < 5 \\ (x-5)^2 & \text{if } x \ge 5 \end{cases}$$

a) $\{a | a \neq \pm 5\}$

b) $\{a | a \neq -5\}$

c) $\{a | a \neq 1\}$

d) $\{a | a \neq \pm 1\}$

e) $\{a | a \neq -1\}$

2. (5 pts.) Which of the following limits is equal to $-\infty$?

a) $\lim_{x\to 4} \dfrac{7-x}{(x-4)^2}$

b) $\lim_{x\to 7^-} \dfrac{x-7}{8}$

c) $\lim_{x\to -5^+} \dfrac{x-9}{x^2(x+5)}$

d) $\lim_{x\to 7^+} \dfrac{8}{x-7}$

e) $\lim_{x\to -5^-} \dfrac{x-9}{x^2(x+5)}$

3. (5 pts.) Which of the following statements is correct?

a) $\lim_{x\to 5} \dfrac{|x-5|}{x-5} = 1$

b) $\lim_{x\to -5^-} \dfrac{|x+5|}{x+5} = 0$

c) $\lim_{x\to -5} |x+5| = 0$

d) $\lim_{x\to -5^-} \dfrac{|x+5|}{x+5} = 5$

e) $\lim_{x\to -5} |x+5| = 5$

4. (5 pts.) Use the given graph of $f(x) = \sqrt{x}$ to find the largest number δ such that $|\sqrt{x} - 1| < \dfrac{1}{2}$ whenever $|x-1| < \delta$.

a) $\dfrac{5}{4}$

b) 1

c) $\dfrac{3}{4}$

d) $\dfrac{1}{2}$

e) $\dfrac{1}{4}$

5. (5 pts.) Given the function

$$f(x) = \begin{cases} x+1 & \text{if } x \le 2 \\ \dfrac{1}{x} & \text{if } 2 < x < 9 \\ \sqrt{x-9} & \text{if } x \ge 9 \end{cases}$$

which of the following statements is true?

a) f is continuous on $(2, \infty)$.

b) f is continuous on $(-\infty, 9)$.

c) f is continuous from the left at $x = 9$.

d) f is continuous from the left at $x = 2$.

e) f has a removable discontinuity at $x = 2$.

6. (5 pts.) Evaluate the limit: $\lim_{x\to 0} x^{10} \cos\left(\dfrac{3}{x}\right)$

a) 3

b) 0

c) The limit does not exist.

d) 1

e) 10

7. (5 pts.) A curve has the equation $y = g(x)$. Choose an expression for the slope of the secant line through the points $P(4, g(4))$ and $Q(x, g(x))$ from the following:

a) $\dfrac{g(x) - g(4)}{4 - x}$

b) $\dfrac{g(x) - g(4)}{x - 4}$

c) $\dfrac{g(4) - g(x)}{x - 4}$

d) $\dfrac{x - 4}{g(x) - g(4)}$

e) $\dfrac{4 + x}{g(4) + g(x)}$

8. (5 pts.) The following limit represents the derivative of some function f at some number a. State such an f and a.

$$\lim_{h\to 0} \frac{\sqrt[3]{343+h} - 7}{h}$$

a) $f(x) = \sqrt[3]{4+x}, a = 338$

b) $f(x) = \sqrt[3]{348+x}, a = 0$

c) $f(x) = \sqrt[3]{x}, a = 343$

d) $f(x) = \sqrt[3]{337+x}, a = \sqrt{6}$

e) $f(x) = \sqrt{337+x}, a = \sqrt{6}$

9. (5 pts.) Examine the graph of the given function f. (Assume that the axes have equal scales.) Select the graph of f'.

10. (5 pts.) Differentiate $F(y) = \dfrac{4-5y}{1-y^2}$.

a) $F'(y) = \dfrac{-(5y^2-8y+5)}{(1-y^2)^2}$

b) $F'(y) = \dfrac{5y^2-8y+5}{(1-y^2)^2}$

c) $F'(y) = \dfrac{-(5y^2-8y+5)}{1-y^2}$

d) $F'(y) = \dfrac{5y^2-8y-5}{1-y^2}$

e) $F'(y) = 0$

11. (5 pts.) If $f(-3) = -3$, $f'(-3) = 4$, $g(-3) = 5$ and $g'(-3) = -4$, find $(fg)'(-3)$.

a) 31

b) 27

c) 38

d) 33

e) 32

12. (5 pts.) The position function of a particle is given by

$$s = t^3 - 7.5t^2 - 9t, \qquad 5 \geq 0$$

When does the particle reach a velocity of 99 $m/\sec$?

a) $t = 7\sec$

b) $t = 1\sec$

c) $t = 9\sec$

d) $t = 4\sec$

e) $t = 7.5\sec$

13. (5 pts.) Find an equation of the tangent line to the curve $y = 5\tan x$ at the point $(\pi/4, 5)$.

a) $y = 5x + 6\left(1 - \dfrac{\pi}{2}\right)$

b) $y = 5x + 13\left(1 - \dfrac{\pi}{2}\right)$

c) $y = 10x + 15\left(1 - \dfrac{\pi}{2}\right)$

d) $y = 12x + 12\left(1 - \dfrac{\pi}{2}\right)$

e) $y = 10x + 5\left(1 - \dfrac{\pi}{2}\right)$

14. (5 pts.) Find the derivative of $y = 9\sin(\sin(x))$ at the point $x = \pi$.

a) 1

b) -9

c) 9

d) -1

e) 0

15. (5 pts.) Find y' by implicit differentiation given $xy = 19 + x^2y$.

a) $\dfrac{x(1+x)}{y(2x+1)}$

b) $\dfrac{y(2x-1)}{x(1-x)}$

c) $\dfrac{x(1-x)}{y(2x-1)}$

d) $\dfrac{x(2y-1)}{1-x}$

e) $\dfrac{y(2x+1)}{x(1+x)}$

16. (5 pts.) The graph of the function g is given below. Use the values in the list below to fill in the blanks in the equations. Each item in the list is used exactly once.

List: (I) doesn't exist (II) 0 (III) 2 (IV) 4 (V) 8

$\lim_{x \to 0} g(x) =$ _____

$\lim_{x \to 2^+} g(x) =$ _____

$g(0) =$ _____

$\lim_{x \to -2^+} g(x) =$ _____

$\lim_{x \to 4^+} g(x) =$ _____

17. (4 pts.) The graphs of four functions are labeled A, B, C, D in the left column. The graphs of their derivatives are labeled I, II, III, IV in the right column.

Fill in the blanks below:

1) The graph of the derivative of A is labeled as _____

2) The graph of the derivative of B is labeled as _____

3) The graph of the derivative of C is labeled as _____

4) The graph of the derivative of D is labeled as _____

18. (5 pts.) Use implicit differentiation to find the slope of the tangent line to the curve
$y^2(y^2 - 4) = x^2(x^2 - 3)$ at the point $(0, -2)$.

19. (6 pts.) (a) State the limit definition of $f'(a)$, the derivative of the function $f(x)$ at the point a.

(b) Use the above definition to find the derivative $f'(x)$ for $f(x) = 3x^2 + 2x - 9$.
(**NO** credit will be given for any other methods.)

20. (5 pts.) Evaluate $\lim_{x \to 2} \dfrac{\sqrt{x+7} - 3}{x - 2}$. Show all your work.