Math 140 Spring 05 EXAMINATION II

1. (5 pts.) If $f(x) = (1-7x)^{-\frac{1}{2}}$, find $f''(0)$.

   a) $f''(0) = \dfrac{-147}{4}$

   b) $f''(0) = \dfrac{49}{2}$

   c) $f''(0) = \dfrac{147}{4}$

   d) $f''(0) = \dfrac{-49}{2}$

   e) $f''(0) = \dfrac{7}{2}$

2. (5 pts.) Find a formula for $f^{(n)}(x)$ where $f(x) = (3+x)^{-1}$.

   a) $f^{(n)}(x) = (-1)^{(n+1)}\dfrac{n!}{n}(3+x)^{-(n+1)}$

   b) $f^{(n)}(x) = (-1)^n n!(3+x)^{-(n+1)}$

   c) $f^{(n)}(x) = (-1)^n \dfrac{n!}{n}(3+x)^{-n}$

   d) $f^{(n)}(x) = (-1)^{(n+1)} n!(3+x)^{-n}$

   e) $f^{(n)}(x) = (-1)^n (n+1)!(3+x)^{-n}$

3. (5 pts.) If $y = 2x^3 + 3x$ and $\dfrac{dx}{dt} = 8$, find $\dfrac{dy}{dt}$ when $x = 5$.

   a) 1225

   b) 1216

   c) 1219

   d) 1236

   e) 1224

4. (5 pts.) The altitude of a triangle is increasing at a rate of 2 cm/min while the area of the triangle is increasing at a rate of 10 cm^2/min. At what rate is the base of the triangle changing when the altitude is 4 cm and the area is 16 cm^2?

   a) 1 cm/min

   b) 2 cm/min

   c) $\dfrac{1}{2}$ cm/min

   d) $\dfrac{1}{4}$ cm/min

   e) 3 cm/min

5. (5 pts.) Find the linearization $L(x)$ of $f(x) = \dfrac{1}{\sqrt{6+x}}$ at $a = 0$.

   a) $L(x) = -\dfrac{1}{\sqrt{6}} + \dfrac{x}{12\sqrt{6}}$

   b) $L(x) = \sqrt{6} - 12\sqrt{6}x$

   c) $L(x) = \dfrac{1}{\sqrt{6}} + \dfrac{x}{12\sqrt{6}}$

   d) $L(x) = \dfrac{1}{\sqrt{6}} - \dfrac{x}{12\sqrt{6}}$

   e) $L(x) = -\dfrac{1}{\sqrt{6}} - \dfrac{x}{12\sqrt{6}}$

6. (5 pts.) Compute Δy and dy for $y = x^2, x = 3$ and $\Delta x = 0.5$.

   a) $\Delta y = 2.25,\ dy = 3$

   b) $\Delta y = 2.25,\ dy = 2$

   c) $\Delta y = 3.25,\ dy = 3$

   d) $\Delta y = 3.25,\ dy = 2$

   e) $\Delta y = 2.5,\ dy = 3$

7. (5 pts.) Find the critical numbers of the function $G(x) = \sqrt[3]{x^2 - 2x}$.

   a) $2, 0$

   b) $2, 4, 0$

   c) $1, 0$

   d) $2, 1, 0$

   e) $-2, -1, 0$

8. (5 pts.) Find the absolute minimum value of $y = 2x^2 + \dfrac{4}{x}$ on the interval $\left[\dfrac{1}{2}, 4\right]$.

   a) 2

   b) 6

   c) 8

   d) 4

   e) 10

9. (5 pts.) Find the number c that satisfies the conclusion of Rolle's Theorem for $f(x) = x\sqrt{x+18}$ in the interval $[-18, 0]$.

   a) $c = -13$

   b) $c = -14$

   c) $c = -12$

   d) $c = 11$

   e) $c = -11$

10. (5 pts.) If $f(3) = 10$ and $f'(x) \geq 4$ for $3 \leq x \leq 7$, how small can $f(7)$ possibly be?

   a) $f(7) = 96$

   b) $f(7) = 33$

   c) $f(7) = 26$

   d) $f(7) = 14$

   e) $f(7) = 10$

11. (5 pts.) Find the inflection points of $f(x) = -2x + 2 - 2\sin x$ on the interval $0 < x < 3\pi$.

   a) $(\pi, -2\pi)$ and $(2\pi, -4\pi + 2)$

   b) $\left(\frac{\pi}{2}, \pi\right)$ and $(\pi, 2\pi - 2)$

   c) $\left(\frac{\pi}{2}, -\pi\right)$ and $(\pi, -2\pi + 2)$

   d) $(\pi, -2\pi + 2)$ and $(2\pi, -4\pi)$

   e) $(\pi, -2\pi + 2)$ and $(2\pi, -4\pi + 2)$

12. (5 pts.) Which of the following is true for the function $f(x) = x^3 - 18x^2 + 96x$?

   a) f is decreasing on $(-\infty, 4)$ and $(8, \infty)$ and increasing on $(4, 8)$

   b) f is increasing on $(-\infty, 4)$ and $(8, \infty)$ and decreasing on $(4, 8)$

   c) f is decreasing on $(-\infty, 12)$ and $(24, \infty)$ and increasing on $(12, 24)$

   d) f is increasing on $(-\infty, 12)$ and $(24, \infty)$ and decreasing on $(12, 24)$

   e) f is increasing on $(-\infty, 8)$ and $(24, \infty)$ and decreasing on $(8, \infty)$

13. (5 pts.) Evaluate the limit $\lim_{x \to \infty} \dfrac{5x + 6}{3x^2 - 2x + 7}$.

   a) ∞

   b) 7

   c) 5

   d) 6

   e) 0

14. (5 pts.) Find the horizontal asymptote of the curve $y = \dfrac{\sqrt[3]{8x^9 + x}}{1 - x^3}$.

   a) $y = 2$

   b) $y = 0$

   c) $y = 8$

   d) $y = -2$

   e) $y = -8$

15. (5 pts.) Which of the following statements is correct?

   a) The graph of $y = \dfrac{x}{(x-1)^2}$ has no vertical asymptote.

   b) The graph of $y = 20x^3 - 3x^5$ is symmetric about the origin.

   c) The graph of $y = \dfrac{x}{x^2 - 9}$ has a horizontal asymptote at $y = 3$.

   d) The graph of $y = \dfrac{x}{x^2 - 9}$ has no horizontal asymptotes.

   e) The graph of $y = \dfrac{x}{x^2 - 9}$ is symmetric about the y-axis.

16. (5 pts.) Find the curve of $y = \dfrac{-21x^2 + 31x - 1}{7x - 1}$.

17. (8 pts.) Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast (in ft/min) is the height of the pile increasing when the pile is 5 ft high? [Hint: $v = \dfrac{1}{3}\pi r^2 h$]

18. (8 pts.) Find the critical numbers of the function $F(x) = x^{\frac{4}{5}}(x-2)^3$. Identify each as a relative maximum, relative minimum or neither.

19. (4 pts.) The figure shows the graphs of three functions. One is the position function $f(x)$ of a car, one is the velocity of the car, and one is its acceleration.
(i) Identify the graph of the velocity function of the car.

Answer a, b or c.

$f' =$ _____

(ii) Identify the graph of the acceleration function of the car.

Answer a, b or c.

$f'' =$ _____