1. Evaluate $\lim_{x\to 0} x^3 \cos\left(\frac{2}{x}\right)$.

   a) The limit does not exist.
   b) 1
   c) 2
   d) 0
   e) ∞

2. Evaluate $\lim_{x\to 4\pi} \sin(x + 3\sin x)$.

   a) 1
   b) -1
   c) 4π
   d) ∞
   e) 0

3. Find the slope of the tangent line to the curve $y = 5x^3$ at the point $(2, 40)$.

   a) $\lim_{h\to 0} \frac{5(2)^3 + 40}{h}$
   b) $\lim_{h\to 0} \frac{5(2^3 - h^3)}{h}$
   c) $\lim_{h\to 0} \frac{5(2+h)^3 - 40}{h}$
   d) $\lim_{h\to 0} \frac{5h^3 - 40}{h}$
   e) The slope does not exist.

4. If the tangent line to $y = f(x)$ at $(8, 9)$ passes through the point $(5, 6)$ find $f'(8)$.

   a) 1
   b) 9
   c) 2
   d) -1
   e) 6

5. If f is a differentiable function, find the derivative of $y = x^4 f(x)$.

   a) $4x^3 f(x) + x^3 f'(x)$
   b) $4x^4 f(x) - x^4 f'(x)$
   c) $3x^3 f(x) - x^4 f'(x)$
   d) $4x^4 f(x) - x^3 f'(x)$
   e) $4x^3 f(x) + x^4 f'(x)$

6. Differentiate $y = \frac{\sin x}{8 + \cos x}$.

   a) $\frac{dy}{dx} = \frac{8\cos x - 9}{(8 + \cos x)^2}$
   b) $\frac{dy}{dx} = \frac{8\cos x + 9}{(8 - \cos x)^2}$
   c) $\frac{dy}{dx} = \frac{8\cos x + 9}{(8 + \cos x)^2}$
   d) $\frac{dy}{dx} = \frac{8\cos x - 1}{(8 - \cos x)^2}$
   e) $\frac{dy}{dx} = \frac{8\cos x + 1}{(8 + \cos x)^2}$

7. If $f(x) = x\sqrt{9 - x^2}$, find $f'(x)$.

   a) $\frac{-9}{x\sqrt{9 - x^2}}$
   b) $\frac{9 - 2x^2}{\sqrt{9 - x^2}}$
   c) $\frac{1}{x\sqrt{9 - x^2}}$
   d) $\frac{-1}{x^2\sqrt{9 - x^2}}$
   e) $\frac{1}{\sqrt{9 - x^2}}$

8. If $x^4 - 7xy + y^3 = -5$, find $\frac{dy}{dx}$ by implicit differentiation at the point $(1, 2)$.

   a) $\frac{1}{2}$
   b) 2
   c) $\frac{3}{5}$
   d) 10
   e) $\frac{25}{11}$

9. If $f(x) = (1 - 3x)^{-\frac{1}{2}}$, find $f''(0)$.

   a) $\frac{9}{2}$
   b) $\frac{3}{2}$
   c) $-\frac{27}{4}$
   d) $\frac{27}{4}$
   e) $-\frac{3}{2}$

10. Find all critical numbers of the function $g(x) = 2x + \sin(2x)$.

a) πn where n is an integer

b) $\frac{\pi}{2}$

c) $\frac{\pi(2n+1)}{4}$ where n is an integer

d) $\frac{\pi(2n+1)}{2}$ where n is an integer

e) π

11. Find the absolute minimum value of $y = -2x^2 + 8x - 5$ on the interval $[0, 5]$.

a) -5

b) 3

c) -15

d) -16

e) 0

12. Find the x-coordinate of the inflection points of the function $f(x) = -6x + 8 - 2\sin x$ in the interval $(0, 3\pi)$.

a) $\pi, 2\pi$

b) $\frac{\pi}{2}, \frac{3\pi}{2}$

c) $\frac{\pi}{4}, \frac{3\pi}{4}$

d) $\frac{3\pi}{2}, \frac{5\pi}{2}$

e) $\frac{7\pi}{2}, \frac{11\pi}{4}$

13. Find $\lim_{x\to\infty} \frac{\sqrt{x^8 - 4}}{\sqrt{x^7 + 4}}$.

a) 0

b) $-\infty$

c) 1

d) ∞

e) -1

14. Find the most general antiderivative of the function $f(x) = 5x^{\frac{1}{4}} - 4x^{\frac{1}{3}}$.

a) $F(x) = 4x^{\frac{5}{4}} - 3x^{\frac{4}{3}} + C$

b) $F(x) = 5x^{\frac{5}{4}} - 4x^{\frac{4}{3}} + C$

c) $F(x) = 4x^{\frac{3}{4}} - 3x^{\frac{2}{3}} + C$

d) $F(x) = \frac{5}{4}x^{-\frac{3}{4}} - \frac{4}{3}x^{-\frac{2}{3}} + C$

e) $F(x) = 5x^4 - 4x^3 + C$

15. Evaluate the integral $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$ by interpreting it in terms of areas.

a) 8π

b) 4π

c) 2π

d) π

e) 4

16. Evaluate the indefinite integral $\int x(10 + 2x^5)dx$.

a) $5x^2 + \frac{2}{7}x^7 + C$

b) $5x^2 + \frac{1}{3}x^6 + C$

c) $10x + \frac{2}{7}x^7 + C$

d) $5x^2 + 2x^7 + C$

e) $\frac{x^2}{2}(10x + 3x^6) + C$

17. Evaluate the integral $\int x^2\sqrt{x^3 + 2}\, dx$.

a) $\frac{1}{9}(x^3 + 2)^{\frac{1}{2}} + C$

b) $-\frac{2}{9}(x^3 + 2)^{\frac{3}{2}} + C$

c) $\frac{2}{9}(x^3 + 2)^{\frac{3}{2}} + C$

d) $\frac{2}{9}(x^3 + 2)^{\frac{1}{2}} + C$

e) $\frac{1}{9}(x^3 - 2)^{\frac{3}{2}} + C$

18. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^2$ and $y = 5$ about the y-axis.

a) $\frac{25}{2}$

b) $\frac{5}{2}$

c) $\frac{25}{2}\pi$

d) 5π

e) 5

19. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating about the $y-$axis the region bounded by the curves $y = \sin x, y = 0, x = 4\pi$ and $x = 7\pi$.

a) $V = \int_0^{4\pi} 2\pi x \sin(x)dx$

b) $V = \int_{4\pi}^{7\pi} 2x \sin(x)dx$

c) $V = \int_{4\pi}^{7\pi} 2\pi \sin(x)dx$

d) $V = \int_{4\pi}^{7\pi} 2\pi x \sin(x)dx$

e) $V = \int_0^{\pi} 2\pi \sin(x)dx$

20. Find the area of the region enclosed by the graphs of $y = 4 - x^2$ and $y = x^2 - 4$.

a) $\frac{64}{3}$

b) $\frac{32}{3}$

c) 64

d) 32

e) 8

21. (10 pts.) If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate (in cm/min) at which the diameter decreases when the diameter is 43 cm. (Hint: Surface area = $4\pi r^2$).

22. (10 pts.) A farmer with 700 feet of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

23. (10 pts.) A region R in the xy-plane is bounded by $y = x^2$ and $y = x$.

a) (2 pts.) Sketch the region R. Be sure to lable the curves and intersection points.

b) (4 pts.) Set up the integral (by using the **Washer Method**) which measures the volume of the solid generated by revolving R around the y-axis. DO NOT evaluate the integral.

c) (4 pts.) Set up the integral (by using the **Cylindrical Shell Method**) which measures the volume of the solid generated by revolving R around the y-axis. DO NOT evaluate the integral.

24. (20 pts.) Consider the function $f(x) = \frac{x^2}{x+8}$ where $f'(x) = \frac{x(x+16)}{(x+8)^2}$ and $f''(x) = \frac{128}{(x+8)^3}$.

(a) (2 pts.) Find the domain of $f(x)$.

(b) (1 pt.) Find the x- and y-intercepts of the graph of $f(x)$.

(c) (1 pt.) Is the graph of $f(x)$ symmetric about the y-axis, symmetric about the origin or neither?

(d) (3 pts.) Find the horizontal, vertical and slant asymptotes for $f(x)$ if any exist.

(e)(3 pts.) Find all the critical numbers for $f(x)$.

(f)(2 pts.) Find the interval(s) where $f(x)$ is increasing and the interval(s) where $f(x)$ is decreasing.

(g)(2 pts.) Find the local minimum and local maximum points of $f(x)$.

(h)(2 pts.) Find the interval(s) where $f(x)$ is concave up and the interval(s) where $f(x)$ is concave down.

(i)(2 pts.) Find all the inflection points for $f(x)$.

(j)(2 pts.) Sketch the graph of $f(x)$.