MATH 141 EXAMINATION II November 10, 2004

Name______________________ ID #______________ Section #_____

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your scantron. On your scantron identify your name, this course (Math 141) and the date. Code and blacken the corresponding circles on your scantron for your student I.D. number and class section number. Code in your **test form**.

There are 13 multiple choice questions worth a total of 65 points. For the problems 1 to 13, either **four** or **five** possible answers are given, only one of which is correct. You should solve the problem, circle the letter of your answer in the exam form and **blacken** the corresponding space on the **scantron**. Mark only one choice; darken the circle completely. There are **7** short-answer and **1** partial credit questions (35 points).

Each of the 7 multiple choice questions $1, 5, 8, 9, 10, 11, 12$ are designed so that partial credit may be given since your answer indicates whether you got part of the question correct.

In order to obtain full credit for the partial credit problem, all work must be shown. Credit will not be given for an answer not supported by work.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 18 PROBLEMS ON 11 PAGES, INCLUDING THIS ONE. CHECK YOUR BOOKLET NOW.

The area below is for the instructor's use.

17. ______________ (12)

18. ______________ (11)

Total ______________ (23)

1. (5 pts.) Determine which of the following expressions are indeterminate forms. (DO NOT compute their limits.)

(i) $\lim_{x\to 0^+} (1+\tan x)^{\frac{1}{x}}$ (ii) $\lim_{x\to 0^+} \dfrac{\sin(2x)}{\ln x}$ (iii) $\lim_{x\to\infty} (\sqrt{x} - \ln x)$

a) Only (i) and (ii) are indeterminate.

b) Only (i) is indeterminate.

c) Only (iii) is indeterminate

d) Only (i) and (iii) are indeterminate.

e) Only (ii) and (iii) are indeterminate.

2. (5 pts.) Compute the value of $\lim_{x\to 0} \dfrac{\tan(\pi x)}{\ln(1+x)}$.

a) 0

b) 1

c) π

d) $-\pi$

e) $+\infty$

3. (5 pts.) Consider the series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

a) The series diverges.

b) The series converges to $\frac{11}{6}$.

c) The series converges to 0.

d) The series converges to $\frac{3}{2}$.

e) None of the above.

4. (5 pts.) Consider the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$.

a) The series converges if $|x| < 2$ to $\frac{2}{2-x}$

b) The series converges if $|x| > 2$ to $\frac{2-x}{2}$

c) The series converges if $|x| < 2$ to $\frac{x}{2-x}$

d) The series converges if $|x| < 2$ to $\frac{2-x}{x}$

e) The series diverges for all x.

5. (5 pts.) Which of the following statements are true?

(i) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$ diverges by *comparison* with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$ diverges by *limit comparison* with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

a) Only (i) is true.

b) Only (ii) is true

c) Both are true.

d) Neither is true.

6. (5 pts.) For what values of p is the infinite series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ convergent? [Hint: Use the integral test and the substitution $u = \ln x$ to integrate.]

a) $p > 1$

b) $p \geq 1$

c) $p > -1$

d) $-1 < p < 1$

e) For all p

7. (5 pts.) Use an improper integral to find the smallest number of terms of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ required to ensure that the the bound obtained for the error in the sum is *less* than 0.005?

a) 110

b) 100

c) 50

d) 11

e) 5

8. (5 pts.) Which of the following two series *converge*?

$$\text{(i)} \sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}, \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{1+e^n}{2^n}.$$

a) Only (i) converges.

b) Only (ii) converges.

c) Both converge.

d) Neither converge.

9. (5 pts.) Which of the following two series *converge*?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ (ii) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3 - 4}$.

a) Only (i) converges.

b) Only (ii) converges.

c) Both converge.

d) Neither converge.

10. (5 pts.) Given the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$, first write out the first few terms in the series and then determine which of the following statements are *true*?

(i) The series diverges by the *test for divergence*.

(ii) The given series diverges by *comparison* with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(iii) The given series converges by the *alternating series test*.

a) None are true.

b) Only (i) is true.

c) Only (ii) is true.

d) Only (iii) is true.

e) Only (i) and (ii) are true.

11. (5 pts.) Which of the following statements are *true*?

(i) The series $\sum_{n=2}^{\infty} \left(\frac{\ln n}{n}\right)^n$ diverges by the *test for divergence.*

(ii) The series $\sum_{n=2}^{\infty} \left(\frac{\ln n}{n}\right)^n$ converges by the *root test.*

(iii) The series $\sum_{n=1}^{\infty} \left(1+\frac{2}{n}\right)^n$ converges by the *root test.*

a) Only (ii) is true.

b) None are true.

c) Only (i) is true.

d) Only (ii) and (iii) are true.

e) Only (iii) is true

12. (5 pts.) Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two series with positive terms, and $\sum_{n=1}^{\infty} a_n$ is convergent. Which of the following statements are **always** true?

(i) $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent.

(ii) If $b_n \geq a_n$ for all positive integers n, then $\sum_{n=1}^{\infty} b_n$ is also convergent.

(iii) $\lim_{n\to\infty} \cos(a_n) = 1$.

a) Only (i) is true.

b) Only (ii) is true.

c) Only (i) and (ii) are true.

d) Only (i) and (iii) are true.

e) All three are ture.

13. (5 pts.) Which of the following statements would allow you to conclude that the series $\sum_{n=1}^{\infty} a_n$ is divergent?

a) $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1.$

b) $a_n \geq \frac{1}{n^2}$ for all n

c) $\lim_{n\to\infty} a_n = 1.$

d) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1.$

e) $a_n \leq \frac{1}{\sqrt{n}}$ for all n

For each series below, determine whether it is absolutely convergent, conditionally convergent, or divergent. Please code, on your answer sheet, **A** if the series is *Absolutely convergent*, **C** if the it is *Conditionally convergent*, or **D** if it is *Divergent*.

14. (4 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$

15. (4 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n 7}{3^n}$

16. (4 pts.) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln(n)}{\ln(n^3)}$

17. (12 pts., 3 pts each) For each of the following, determine if the **sequence** $\{a_n\}_{n=1}^{\infty}$ converges or diverges. If it converges find the limit of the sequence. Please circle your answers.

a) $a_n = \dfrac{(n+1)!}{(n-1)!n^2}$

Divergent or **Converges** to ________

b) $a_n = \dfrac{(-1)^{n-1}n}{n^3+1}$

Divergent or **Converges** to ________

c) $a_n = \sqrt{n} - \sqrt{n+1}$

Divergent or **Converges** to ________

d) $a_n = (e-1)^n$

Divergent or **Converges** to ________

18. (11 pts.) Compute the integral

$$\int_0^2 \frac{dx}{(x-1)^2}.$$

For full credit, show all your work and circle/fill in the appropriate line below with your final answer.

The integral *converges* to __________ or the integral *diverges*.