

MATH 141
EXAMINATION II
SPRING 2003

NAME _____
STUDENT NUMBER _____
INSTRUCTOR _____
SECTION NUMBER _____

This examination consists of 12 problems. The first 6 are multiple choice questions, the next two are short answer questions and the remaining 4 are partial credit problems. The point value for each question appears to the left of the question number. There are 100 total points.

Please record your answers to the multiple choice questions by circling the corresponding letter. Present your work clearly for the partial credit problems. **No credit will be given for unsupported answers.**

THE USE OF CALCULATORS, BOOKS, NOTES ETC. DURING THIS EXAMINATION IS PROHIBITED.

**Check the examination booklet before you start.
There should be 12 problems on 10 pages.**

DO NOT WRITE IN THE BLANKS BELOW.

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|-------------|---------------|
| 1. _____(5) | 7. _____(15) |
| 2. _____(5) | 8. _____(15) |
| 3. _____(5) | 9. _____(10) |
| 4. _____(5) | 10. _____(10) |
| 5. _____(5) | 11. _____(10) |
| 6. _____(5) | 12. _____(10) |

SUBTOTAL _____

TOTAL _____

- 5 pts 1. Determine whether the improper integral is convergent or divergent. If it is convergent, find its value.

$$\int_0^{\infty} x e^{-x^2} dx$$

- a) The integral converges to $-\frac{1}{2}$
- b) The integral converges to 0
- c) The integral converges to $\frac{1}{4}$
- d) The integral converges to $\frac{1}{2}$
- e) The integral diverges

- 5 pts 2. Find the limit of the sequence

$$\left\{ \frac{9^{n+1} + (-1)^n}{(3^n + n)^2} \right\}.$$

- a) 0
- b) 1
- c) 3
- d) 9
- e) The sequence diverges

- 5 pts 3. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{2^{2n}}$$

- a) The series converges to -3
- b) The series converges to $-\frac{3}{7}$
- c) The series converges to $\frac{4}{7}$
- d) The series converges to 4
- e) The series diverges

- 5 pts 4. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$$

- a) The series converges to -1
- b) The series converges to 0
- c) The series converges to $e - 1$
- d) The series converges to e
- e) The series diverges

5 pts 5. If $\sum_{n=1}^{\infty} a_n = 2$, what is $\lim_{n \rightarrow \infty} (1 + a_n)$?

a) 0

b) 1

c) 2

d) 3

e) The limit does not exist

5 pts 6. If $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, find $\sum_{n=3}^{\infty} \frac{1}{n^2}$

a) $\frac{\pi^2}{6}$

b) $\frac{\pi^2}{6} - 1$

c) $\frac{\pi^2}{6} - \frac{5}{4}$

d) $\frac{\pi^2}{6} + \frac{5}{4}$

e) The series diverges

- 15 pts 7. For each sequence below, circle the correct answer. If you circle answer a), you must complete the sentence with the limit of the sequence to receive credit. No partial credit will be given.

I) $\left\{ \frac{\sin n}{n} \right\}$

- a) The sequence converges to _____.
b) The sequence diverges.

II) $\left\{ \frac{e^{\ln n}}{2n} \right\}$

- a) The sequence converges to _____.
b) The sequence diverges.

III) $\{\ln(2n) - \ln(n + 1)\}$

- a) The sequence converges to _____.
b) The sequence diverges.

IV) $\left\{ \frac{n}{(\ln n)^2} \right\}$

- a) The sequence converges to _____.
b) The sequence diverges.

V) $\left\{ \frac{3n^3 + \ln n}{7n^3 + 2 \ln n} \right\}$

- a) The sequence converges to _____.
b) The sequence diverges.

15 pts 8. For each series below, circle the correct answer. No partial credit will be given.

$$\text{I) } \sum_{n=1}^{\infty} \frac{1}{n}$$

ABS. CONV.

COND. CONV.

DIVERGENT

$$\text{II) } \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

ABS. CONV.

COND. CONV.

DIVERGENT

$$\text{III) } \sum_{n=1}^{\infty} \tan^{-1} n$$

ABS. CONV.

COND. CONV.

DIVERGENT

$$\text{IV) } \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

ABS. CONV.

COND. CONV.

DIVERGENT

$$\text{V) } \sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

ABS. CONV.

COND. CONV.

DIVERGENT

- 10 pts 9. Determine whether the following series converges absolutely, converges conditionally or diverges. Indicate the theorem(s) and/or test(s) used and justify why they are applicable. Full justification is required, in particular the logic of your argument must be made clear. Unjustified answers will receive no credit regardless of their correctness.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

- 10 pts 10. Determine whether the following series converges or diverges. Indicate the theorem(s) and/or test(s) used and justify why they are applicable. Full justification is required, in particular the logic of your argument must be made clear. Unjustified answers will receive no credit regardless of their correctness.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

- 10 pts 11. Determine whether the following series converges or diverges. Indicate the theorem(s) and/or test(s) used and justify why they are applicable. Full justification is required, in particular the logic of your argument must be made clear. Unjustified answers will receive no credit regardless of their correctness.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+2003}}{(n+5\sin n)^2}$$

- 10 pts 12. Determine whether the following series converges or diverges. Indicate the theorem(s) and/or test(s) used and justify why they are applicable. Full justification is required, in particular the logic of your argument must be made clear. Unjustified answers will receive no credit regardless of their correctness.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{7^n \cdot (n!)^2}$$

Math 141 Spring 2003 Exam 2 Answers

1.d, 2.d, 3.b, 4.c, 5.b, 6.c

7.(I) converges to 0, (II) converges to $\frac{1}{2}$, (III) converges to $\ln 2$, (IV) diverges,
(V) converges to $\frac{3}{7}$.

8.(I) divergent, (II) conditionally convergent, (III) divergent, (IV) absolutely convergent,
(V) absolutely convergent.

9. Converges conditionally by the alternating series test and the (direct) comparison test.

10. Diverges by the integral test.

11. Converges by the limit comparison test.

12. Converges by the ratio test.