

MATH 141

NAME _____

EXAMINATION II

STUDENT NUMBER _____

MARCH 24, 2004

INSTRUCTOR _____

TEST FORM A

SECTION NUMBER _____

This examination consists of 12 problems. The first 6 are multiple choice questions, the next two are short answer questions and the remaining 4 are partial credit problems. The point value for each question appears to the left of the question number. There are 100 total points.

Please record your answers to the multiple choice questions by circling the corresponding letter. Present your work clearly for the partial credit problems. **No credit will be given for unsupported answers.**

THE USE OF CALCULATORS, BOOKS, NOTES ETC.

DURING THIS EXAMINATION IS PROHIBITED.

Do not write in the blanks below.

1. _____(5)

7. _____(12)

2. _____(5)

8. _____(10)

3. _____(5)

9. _____(12)

4. _____(5)

10. _____(8)

5. _____(5)

11. _____(8)

6. _____(5)

12. _____(10)

SUBTOTAL _____

TOTAL _____

5 pts 1. Find the sum of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{2 \cdot 4^n}$

- a) $\frac{1}{24}$
- b) $-\frac{1}{10}$
- c) $\frac{1}{15}$
- d) $\frac{1}{40}$
- e) The series diverges

5 pts 2. If $a_n = \left(\frac{\sin^2 n}{2}\right)^n$, then the sequence $\{a_n\}$

- a) diverges by oscillation
- b) converges to 0
- c) converges to 1
- d) converges to $\frac{\pi}{2}$
- e) diverges to $+\infty$

5 pts 3. The series

$$\sum_{n=1}^{\infty} \left(\tan^{-1} n - \tan^{-1}(n+1) \right)$$

converges to

- a) $-\frac{\pi}{4}$
- b) 1
- c) 0
- d) $-\frac{\pi}{2}$
- e) It does not converge, it diverges to $-\infty$.

5 pts

4. When evaluated, the integral $\int_0^{+\infty} x e^{-x} dx$

- a) converges to 0.
- b) converges to 1.
- c) converges to e .
- d) diverges to $-\infty$.
- e) diverges to $+\infty$.

5 pts 5. Given the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!},$$

determine the smallest n that will allow s_n , the n -th partial sum, to approximate the actual sum to within $\frac{1}{100}$.

- a) 2
- b) 4
- c) 6
- d) 8
- e) 10

5 pts

6. The series $\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 1}$

- a) Converges by the Direct Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.
- b) converges by the n th term Test for Divergence.
- c) converges by the n th Root Test.
- d) Diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.
- e) diverges by the Ratio Test.

12 pts 7. If $\sum a_n$ is a convergent series with all positive terms, i.e., $a_n > 0$ for all n , then determine which of the following must always be true. Circle the correct answer that corresponds to each question. NOTE: You do not need to justify your work.

3 pts a) $\sum \cos a_n$ Must converge Must diverge Can not be determined

3 pts b) $\sum \frac{1}{a_n}$ Must converge Must diverge Can not be determined

3 pts c) $\sum \sqrt{a_n}$ Must converge Must diverge Can not be determined

3 pts d) $\sum (a_n)^2$ Must converge Must diverge Can not be determined

- 10 pts 8. Determine whether the following sequences converge or diverge. For credit, circle your answer choice and find the limit of each convergent sequence. NOTE: You do not need to show your work.

2 pts a) $\left\{ \frac{\sqrt{3n^2 + 2}}{2n + 3} \right\}$ Diverges or Converges to _____

2 pts b) $\left\{ \frac{2}{3}, \frac{-4}{5}, \frac{6}{7}, \frac{-8}{9}, \dots, \frac{(-1)^n \cdot 2n}{2n + 1}, \dots \right\}$ Diverges or Converges to _____

2 pts c) $\left\{ \frac{n!}{n^n} \right\}$ Diverges or Converges to _____

2 pts d) $\left\{ \cos \frac{5}{n} \right\}$ Diverges or Converges to _____

2 pts e) $\left\{ n^{\left(\frac{1}{n}\right)} \right\}$ Diverges or Converges to _____

12 pts 9. Determine whether the following series converge absolutely, converge conditionally, or diverge. Circle the correct answer that corresponds to each question. NOTE: You do not need to justify your work.

3 pts a) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ Absolutely converges Conditionally converges Diverges

3 pts b) $\sum_{n=0}^{\infty} \frac{n!}{(-2)^n}$ Absolutely converges Conditionally converges Diverges

3 pts c) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n-1}}$ Absolutely converges Conditionally converges Diverges

3 pts d) $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{2n}$ Absolutely converges Conditionally converges Diverges

10. Evaluate $\int_0^4 \frac{1}{(x-2)^3} dx$. For credit, you must show all work that leads to your answer.

8 pts

8 pts 11. Test the series

$$\sum_{n=4}^{\infty} \frac{n^3 \sqrt{n} - 1}{3n^4 + 5\sqrt{n} + 1}$$

for convergence or divergence. At the bottom of the page, name the tests or theorems used and fully justify your conclusion. **No credit will be given for unsupported answers.**

Test(s) or theorem(s) used _____

Conclusion _____

10 pts 12. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$$

converges absolutely, converges conditionally, or diverges. At the bottom of the page, name the tests or theorems used and fully justify your conclusion. **No credit will be given for unsupported answers.** You must give a full justification of your answer.

Test(s) or theorem(s) used _____

Conclusion _____