MATH 141

FINAL EXAM

DECEMBER 17, 2003

NAME _____

STUDENT NUMBER _____

INSTRUCTOR _____

SECTION NUMBER _____

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your scantron. On your scantron identify your name, this course (Math 141) and the date. Code and blacken the corresponding circles on your scantron for your student I.D. number and class section number.

There are 14 multiple choice questions worth a total of 72 points. Problems 1 to 10 count 6 points each. Problems 11-14 count 3 points each. For the problems 1 to 10 **five** possible answers are given, only one of which is correct. For the problems 11 to 14 **three** possible answers are given, only one of which is correct. You should solve the problem, circle the letter of your answer in the exam form and **blacken** the corresponding space on the **scantron**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test is the same as the problem number of the scantron. There are **5** partial credit questions (78 points). **In order to obtain full credit for the partial credit problems, all work must be shown**. **Credit will not be given for an answer not supported by work**. The point value for each partial credit question is given in parentheses to the right of the question number.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

15. (15 pts.) _____	**Do not write in the box to the left.**
16. (15 pts.) _____	
17. (15 pts.) _____	
18. (15 pts.) _____	
19. (18 pts.) _____	
Total _____	

1. (6 pts.) Let $f(x) = \sin^{-1}(x) + \cos^{-1}(x)$, where $|x| \leq 1$. $f'(\pi/4)=$

a) 0

b) 1

c) $\dfrac{2}{(4-\pi^2)^{1/2}}$

d) $\dfrac{4}{(4-\pi^2)^{1/2}}$

e) 2

2. (6 pts.) Let $f(x) = (\tan x)^x$. Then $f'(\pi/4)$ is equal to

a) $\pi/4$

b) $\pi/8$

c) ∞

d) π

e) $\pi/2$

3. (6 pts.) The interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{n2^n}$$

is:

a) $[-2, 2]$

b) $(-2, 2)$

c) $(-2, 2]$

d) $[-2, 2)$

e) $(-1, 1)$

4. (6 pts.) The sequence $\left\{n^{3/n}\right\}_{n=1}^{\infty}$

a) converges to 1

b) converges to 3

c) diverges

d) converges to 0

e) converges to $\frac{1}{3}$

5. (6 pts.) The power series expansion for $\int \frac{1}{1+x^3}\,dx$ is

a) $C + x - \frac{x^4}{4} + \frac{x^7}{7} - \frac{x^{10}}{10} \cdots = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{3n+1}$

b) $C + x + \frac{x^4}{4} + \frac{x^7}{7} + \frac{x^{10}}{10} \cdots = C + \sum_{n=0}^{\infty} \frac{x^{3n+1}}{3n+1}$

c) $C + 1 + x^3 + x^6 + x^9 \cdots = C + \sum_{n=0}^{\infty} x^{3n}$

d) $C + 1 - x^3 + x^6 - x^9 \cdots = C + \sum_{n=0}^{\infty} (-1)^n x^{3n}$

e) $C + 3x^2 + 6x^5 + 9x^8 \cdots = C + \sum_{n=1}^{\infty} 3nx^{3n-1}$

6. (6 pts.) Let $T_3(x)$ be the Taylor polynomial of degree 3 centered at $x = 0$ for $f(x) = xe^x$. Then $T_3(2) =$

a) 14

b) 10

c) 6

d) $\frac{38}{3}$

e) 8

7. (6 pts.) The series $\displaystyle\sum_{n=17}^{\infty} \frac{1}{n \ln n}$

   a) converges by the integral test.

   b) diverges by the integral test.

   c) diverges by comparison with $\displaystyle\sum_{n=17}^{\infty} \frac{1}{n}$

   d) converges by comparison with $\displaystyle\sum_{n=17}^{\infty} \frac{1}{n^2}$

   e) diverges by the limit comparison test, compared with $\displaystyle\sum_{n=17}^{\infty} \frac{1}{n}$

8. (6 pts.) If we apply the ratio or the root test to a p-series, the limit we obtain will always be:

   a) The first term of the series.

   b) The sum of the series if it is convergent, infinity if it is divergent.

   c) The number p.

   d) We will always get 1, thus the ratio and root tests are always inconclusive for a p-series.

   e) None of the above.

9. (6 pts.) A polar equation for the curve $x^2 - y^2 = \sqrt{x^2 + y^2}$ is:

a) $r = \cos(2\theta)$

b) $r = \sin(2\theta)$

c) $r\cos(2\theta) = 1.$

d) $r\sin(2\theta) = 1.$

e) $\sin(2\theta) = 1$

10. (6 pts.) Which of following is the graph for the polar equation $r = 5 + 3\sin 3\theta$

11. (3 pts.) Given a series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n\to\infty} a_n$ does not exist, then :

a) The series must diverge.

b) No conclusion can be reached concerning the convergence or divergence of the series.

c) The series must converge.

12. (3 pts.) Given a series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then:

a) The series must diverge.

b) No conclusion can be reached concerning the convergence or divergence of the series.

c) The series must converge.

13. (3 pts.) Given a series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \frac{99}{100}$, then :

a) The series must diverge.

b) No conclusion can be reached concerning the convergence or divergence of the series.

c) The series must converge.

14. (3 pts.) Given a series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n\to\infty} a_n = 0$, then:

a) The series must diverge.

b) No conclusion can be reached concerning the convergence or divergence of the series.

c) The series must converge.

15. (15 points) Evaluate the definite integral

$$\int_0^{\pi} x^2 \cos x \; dx.$$

16. (15 points) Find the length of the curve given parametrically by

$$x(t) = 2e^{-t}(\sin(t) + \cos(t)) \ , \ y(t) = 2e^{-t}(\sin(t) - \cos(t)) \ , \ 0 \ \leq \ t \ \leq \ 4.$$

17. (15 points) Determine whether the following series are absolutely convergent, conditionally convergent or divergent. Justify your answers and give the name(s) of any test(s) you used to reach your conclusions.

a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)^{1/3}}$

b) (5 points) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^3+7}{3n^3+6}\right)^n$

c) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$

18. (15 points) Evaluate the **improper integral** $\displaystyle\int_{-2}^{2} \frac{1}{(4-x^2)^{1/2}}\,dx$**.**

19. (18 points)

a) (12 points) Write down the Taylor series for the function $f(x) = \sin(2x)$ centered at $x = \pi/2$.

b) (3 points) What is $T_3(\frac{\pi+1}{2})$ (the Taylor polynomial of degree 3 evaluated at $\frac{\pi+1}{2}$)?

c) (3 points) Estimate the error $\left|\sin(\pi+1) - T_3(\frac{\pi+1}{2})\right|$.

---SECTION 5 - Item analysis data

NUMBER OF STUDENTS IN ITEM ANALYSIS 577 NO. OF ITEMS: 14

The RESPONSE TABLE below contains the percentages of students who selected each option to each item. The percentage selecting the correct option is repeated in the KEY - % column. The ITEM EFFECT column contains biserial coefficients. Negative values indicate ineffective items; some students with high test scores missed these items. A "+" sign identifies an incorrect option selected by students with a higher average test score than those who selected the correct option.

RESPONSE TABLE - FORM A

ITEM NO.	OMIT %	A %	B %	C %	D %	E %	KEY	- %	ITEM EFFECT
1	0	89	2	6	2	1	A	89	0.44
2	0	13	11	7	4	65	E	65	0.37
3	0	8	19	7	63	2	D	63	0.48
4	0	66	6	14	13	1	A	66	0.31
5	0	54	15	8	20	2	A	54	0.30
6	0	5	41	13	23	18	B	41	0.26
7	0	14	64	10	4	8	B	64	0.46
8	0	5	5	3	75	11	D	75	0.36
9	0	13	5	67	12	3	C	67	0.32
10	0+	4	76	4	9	7	B	76	0.29
11	0	88	11	1	0	0	A	88	0.43
12	0	8	86	6	0	0	B	86	0.45
13	0	5	2	92	0	0	C	92	0.44
14	0	3	71	25	0+	0	B	71	0.45

Distribution of items by % CORRECT:

% Correct		No. of items
0 - 20	(very difficult)	0
21 - 60	(difficult)	2
61 - 90	(moderately difficult)	11
91 - 100	(easy)	1

Distribution of items by ITEM EFFECT:

Biserial Coefficient		No. of items
Negative	(ineffective)	0
.00 - .20	(low effectiveness)	0
.21 - .40	(medium effectiveness)	7
.41 - 1.00	(high effectiveness)	7

The RELIABILITY of scores for this test is 0.679

The reliability coefficient above is a Kuder-Richardson formula 20 value. K-R 20 values range from .00 to 1.00. Higher values indicate greater measurement precision. Scores on tests with K-R 20 values below .50 are poor indicators of which students are strong and which are weak on the information tested.