MATH 141 NAME _____

FINAL EXAMINATION STUDENT NUMBER _____

DECEMBER 15, 2004 INSTRUCTOR _____

FORM A SECTION NUMBER _____

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your scantron. On your scantron identify your name, this course (Math 141) and the date. Code and blacken the corresponding circles on your scantron for your student I.D. number and class section number. Code in your **test form**.

There are 20 multiple choice questions worth a total of 100 points. For problems 1 to 20, **five** possible answers are given, only one of which is correct. There are **3** short-answer/multiple choice questions (21 to 23) with **three** possible answers each, worth a total of 15 points. There are **4** matching questions, 24 to 27, worth **2** points each. You should solve each problem, circle the letter of your answer in the exam form and **blacken** the corresponding space on the **scantron**. Mark only one choice; darken the circle completely. There are **2** partial credit questions (27 points).

Each of the 5 multiple choice questions $7, 8, 9, 10, 11$ are designed so that partial credit may be given since your answer indicates whether you got part of the question correct.

**In order to obtain full credit for the partial credit problems, all work must be shown**. **Credit will not be given for an answer not supported by work**.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 29 PROBLEMS ON 15 PAGES, INCLUDING THIS ONE.
CHECK YOUR BOOKLET NOW.

**The area below is for the instructor's use.**

28. _____ (12)

29. _____ (15)

Total _____ (27)

1. (5 pts.) Evaluate $\displaystyle\int_a^b 5^x\,dx, \qquad b > a > 0.$

   a) $\dfrac{1}{\ln 5}(5^b - 5^a)$

   b) $5^{(b-a)}$

   c) $5^b - 5^a$

   d) $\dfrac{a}{b}$

   e) $b^5 - a^5$

2. (5 pts) $4^{(\log_2 9 + \log_2 4 - \log_2 6)} =$

   a) $4^6$

   b) $4^7$

   c) 14

   d) 49

   e) 36

3. (5 pts) Suppose $f(x) = x^{\tan x}$, find $f'(x)$.

a) $x^{\tan x}$

b) $x^{\tan x} \sec^2 x$

c) $(\tan x - 1)x^{\tan x}$

d) $\dfrac{\tan x}{x} + \sec^2 x \, \ln x$

e) $\left(\dfrac{\tan x}{x} + \sec^2 x \, \ln x\right) x^{\tan x}$

4. (5 pts) Evaluate $\displaystyle\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\, dx$.

a) $-\pi$

b) $-\dfrac{\pi}{4}$

c) $0$

d) $\dfrac{\pi}{6}$

e) $\dfrac{\pi}{3}$

5. (5 pts) When using partial fractions decomposition to simplify $\dfrac{1}{x^2(x^2-1)(x^2+4)}$, the correct starting expression to use is

a) $\dfrac{a}{x}+\dfrac{b}{x^2}+\dfrac{c}{x+1}+\dfrac{d}{x-1}+\dfrac{ex+f}{x^2+4}$.

b) $\dfrac{a}{x}+\dfrac{b}{x^2}+\dfrac{cx+d}{x^2-1}+\dfrac{ex+f}{x^2+4}$.

c) $\dfrac{ax+b}{x^2}+\dfrac{c}{x+1}+\dfrac{d}{x-1}+\dfrac{ex+f}{x^2+4}$.

d) $\dfrac{a}{x}+\dfrac{b}{x^2}+\dfrac{c}{x+1}+\dfrac{d}{x-1}+\dfrac{e}{x^2+4}$.

e) $\dfrac{ax+b}{x^2}+\dfrac{cx+d}{x^2-1}+\dfrac{ex+f}{x^2+4}$.

6. (5 pts) Evaluate $\displaystyle\int_0^{\pi/2} \sin^2 t \cos^3 t \, dt$.

a) 0

b) $-2$

c) $-\dfrac{1}{15}$

d) $\dfrac{2}{15}$

e) $\dfrac{8}{15}$

7. (5 pts.) Determine which of the following expressions are indeterminate forms. (DO NOT compute the limits.)

(i) $\lim_{x\to 0} \dfrac{x}{e^{1/x^2}}$ (ii) $\lim_{x\to \frac{\pi}{2}} \left(x - \dfrac{\pi}{2}\right) \tan x$ (iii) $\lim_{x\to\infty} \left(\dfrac{1}{x}\right)^{\sqrt{x}}$

a) Only (ii) and (iii) are indeterminate.

b) Only (i) is indeterminate.

c) Only (ii) is indeterminate

d) All are indeterminate.

e) None are indeterminate.

8. (5 pts.) Consider the series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{1+c}\right)^n$, where $c \neq -1$. Which of the following statements are *true*?

(i) The series is geometric for every choice of $c \neq -1$.
(ii) The series converges for $c = -\dfrac{1}{3}$.
(iii) The series converges to 2 when $c = 1$.

a) Only (i) and (iii) are true.

b) Only (i) and (ii) are true.

c) Only (i) is true.

d) None are true.

e) All are true.

9. (5 pts.) Given the series $\sum_{n=3}^{\infty}\left[\frac{1}{n+1}+\frac{1}{n-2}\right]$, which of the following statements are *true*?

(i) The series diverges by the *test for divergence*.
(ii) The series converges by the *integral test*.
(iii) The series diverges using the *limit comparison test*.

a) Only (i) is true.

b) Only (iii) is true.

c) Only (i) and (ii) are true.

d) Only (ii) is true.

e) None are true.

10. (5 pts.) Given the series $\sum_{n=1}^{\infty}\frac{2^n}{n^n}$, which of the following statements are *true*?

(i) The series diverges by the *test for divergence*.
(ii) The series converges by the *root test*.
(iii) The series converges by the *ratio test*. Hint: Use $e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

a) Only (ii) is true.

b) Only (i) and (ii) are true.

c) Only (i) and (iii) are true.

d) All are true.

e) Only (ii) and (iii) are true.

11. (5 pts.) Which of the following statements are *true*?

(i) The series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt[4]{n^3+1}}$ diverges by the *limit comparison test.*

(ii) The series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^4+2n^2}$ converges by the *limit comparison test.*

(iii) The series $\displaystyle\sum_{n=1}^{\infty} (-1)^{2n}$ converges.

a) Only (i) is true.

b) Only (i) and (iii) are true.

c) All are true.

d) Only (i) and (ii) are true.

e) None are true.

12. (5 pts.) The interval of convergence for the power series

$$\sum_{n=0}^{\infty} (-1)^n n 3^n x^n$$

is

a) $\{0\}$

b) $\left(-\frac{1}{3}, \frac{1}{3}\right)$

c) $\left[-\frac{1}{3}, \frac{1}{3}\right)$

d) $\left(-\frac{1}{3}, \frac{1}{3}\right]$

e) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

13. (5 pts.) Which of the following is a power series representation for $\dfrac{5}{1+2x^3}$?

a) $\displaystyle\sum_{n=0}^{\infty} -10x^{3n} = 1 - 10x^3 - 10x^6 - 10x^9 - ...$

b) $\displaystyle\sum_{n=0}^{\infty} 5(-2)^n x^{3n} = 5 - 10x^3 + 20x^6 - 40x^9 + ...$

c) $\displaystyle\sum_{n=0}^{\infty} 5(-2x)^{3n} = 5 - 40x^3 + 320x^6 - 2560x^9 + ...$

d) $\displaystyle\sum_{n=0}^{\infty} -5(2^n)x^{3n} = -5 - 10x^3 - 20x^6 - 40x^9 - ...$

e) $\displaystyle\sum_{n=0}^{\infty} 5(-2)^n x^{3+n} = 5x^3 - 10x^4 + 20x^5 - 40x^6 + ...$

14. (5 pts.) What is the area of the region enclosed by the curve $r = \dfrac{2}{3}\sin 2\theta$?

a) $0$

b) $\dfrac{8\pi - 1}{36}$

c) $\dfrac{2\pi - 4}{9}$

d) $\dfrac{4\pi}{9}$

e) $\dfrac{2\pi}{9}$

15. (5 pts.) Which of the following is the Maclaurin series for $f(x) = \cos(2x)$?

a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2x)^n}{n!}$

b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2x)^{2n}}{(2n)!}$

c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{2x^{2n}}{(2n)!}$

d) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(2x)^n}{n!}$

e) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(2x)^{2n}}{(2n)!}$

16. (5 pts.) Consider the following parametric equations.

$$x = \frac{1}{2}t^2 - 2t + 1$$
$$y = 3t^4 - 8t^3 + 4$$

For what values of $t$ is the graph of $(x, y)$ concave upward?

a) $(-\infty, 0)$

b) $(0, 2)$

c) $(2, \infty)$

d) $(-\infty, 2)$

e) $(-\infty, 0) \cup (2, \infty)$

17. (5 pts.) Let $x = t^2 + 1$ and $y = 2t^3 - 3t$. An equation for the tangent line to the curve at $t = 1$ is

a) $y = \frac{3}{2}x + 2$

b) $y = \frac{2}{3}x - \frac{7}{3}$

c) $y = -\frac{1}{2}x$

d) $y = \frac{3}{2}x - 4$

e) $y = -2x + 3$

18. (5 pts) Determine the limit of the **sequence** $\left\{\frac{\sin n}{n}\right\}_{n=1}^{\infty}$.

a) 0

b) 1

c) $+\infty$

d) $-\infty$

e) Does not exist, but is neither $-\infty$ nor $+\infty$.

19. (5 pts) Determine the limit of the **sequence** $\left\{\dfrac{3^n+1}{2^n+1}\right\}_{n=1}^{\infty}$.

a) 0

b) 1

c) 2/3

d) $+\infty$

e) Does not exist, but is not $+\infty$.

20. (5 pts) Determine the limit of the **sequence** $\left\{\dfrac{-n+1}{\sqrt{n}}\right\}_{n=1}^{\infty}$.

a) 0

b) 1

c) $+\infty$

d) $-\infty$

e) Does not exist, but is neither $-\infty$ nor $+\infty$.

For each series below, determine whether it is absolutely convergent, conditionally convergent, or divergent. Please code, on your answer sheet, **A** if the series is *Absolutely convergent*, **C** if the it is *Conditionally convergent*, or **D** if it is *Divergent*.

21. (5 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+2) 4^n}{5^{n+1}}$

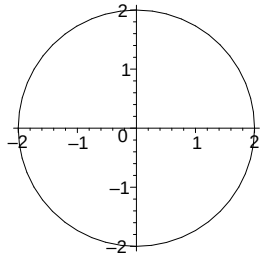
22. (5 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{2}}{n^{1/4}}$

23. (5 pts.) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{n}{3n+1}\right)$

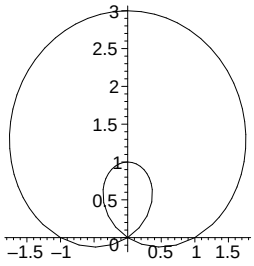
For questions 24-27, you are given 5 equations, labeled a) through e), and 4 graphs, labeled I through IV. There is only one equation that matches each graph. **Code your answers on the answer sheet.**

a) $x = 2\cos t,\ y = 2\sin t$
b) $x = \cos t,\ y = \cos^2 t$
c) $r^2 \sin(2\theta) = 1$
d) $r = \cos\theta$
e) $r = 1 + 2\sin\theta$

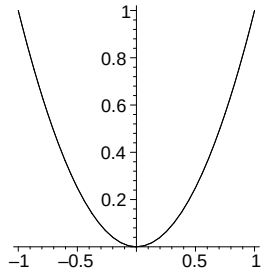
I.



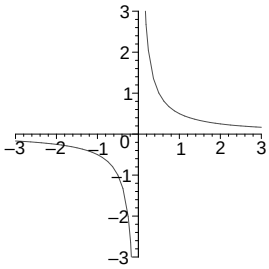

II.




III.




IV.




24. (2 pts) Which equation corresponds to the graph I?

25. (2 pts) Which equation corresponds to the graph II?

26. (2 pts) Which equation corresponds to the graph III?

27. (2 pts) Which equation corresponds to the graph IV?

28. (12 pts) Find

$$\int_0^{\pi} x \sin 2x \, dx$$

29. (15 pts) Let $f(x) = e^{3x}$.

a) Find $f'(0), f''(0)$, and $f'''(0)$. Based on your answers, what is the general formula for $f^{(n)}(0)$, its $n$-th derivative evaluated at $x = 0$?

b) Use your answer in part a) to write down the Maclaurin series for $f(x)$ in summation notation.

c) Find the radius of convergence for the Maclaurin series found in part b).