MATH 141 FINAL EXAMINATION Spring 2003

Name______________________________ ID #__________________ Section #______

Instructor ________________________

There are 14 multiple choice, 2 short-answer, and 5 partial credit type questions. To obtain credit for all partial credit problems all work must be shown. The examination is worth 150 points. The point value of each problem is shown next to the problem number.

The examination papers will be collected at the end of the examination.

THE USE OF CALCULATORS, BOOKS, NOTES, ETC., IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 21 PROBLEMS ON 15 PAGES, INCLUDING THIS ONE. CHECK YOUR BOOKLET NOW.

The box below is for the instructor's use.

MC .... (70)

Short Ans......... (18)

17 .... (10)

18 .... (10)

19 .... (14)

20 .... (14)

21 .... (14)

Total .... (150)

5 pts 1. Order the functions e^x, x^3, and $(\ln x)^4$ from **the Slowest** growing to **the Fastest** growing as $x \to \infty$.

a) $(\ln x)^4,\ x^3,\ e^x$

b) $e^x,\ (\ln x)^4,\ x^3$

c) $x^3,\ e^x,\ (\ln x)^4$

d) $x^3,\ (\ln x)^4,\ e^x$

e) $(\ln x)^4,\ e^x,\ x^3$

5 pts 2. Evaluate

$$A = \log_3 81, \text{ and } B = e^{2\ln 4}.$$

a) $A = 4,\ B = 8$

b) $A = 2,\ B = 16$

c) $A = 5,\ B = 32$

d) $A = 5,\ B = 8$

e) $A = 4,\ B = 16$

5 pts 3. Let $f(x) = (\cos x)^x,\ 0 \le x \le \frac{\pi}{2}$. Find $f'(x)$.

a) $x(\cos x)^{x-1}$

b) $-(\sin x)(\ln \cos x)(\cos x)^x$

c) $(\cos x)^x \left(\ln \cos x - \frac{x}{\cos x}\right)$

d) $(\cos x)^x(\ln \cos x + x \tan x)$

e) $-x \sin x (\cos x)^{x-1}$

5 pts 4. The function $f(x) = 3x + x^3 - 2$ is known to have a differentiable inverse. Find $(f^{-1})'(2)$.

a) 1

b) $\frac{1}{12}$

c) $\frac{1}{2}$

d) $\frac{1}{15}$

e) $\frac{1}{6}$

5 pts 5. Evaluate

$$\int \frac{6}{(2x-1)(2x+1)}\,dx.$$

a) $6\ln|2x-1|\cdot\ln|2x+1|+C$

b) $\frac{3}{2}\ln\left|\frac{2x-1}{2x+1}\right|+C$

c) $3\ln\left|\frac{2x-1}{2x+1}\right|+C$

d) $3\ln|(2x-1)(2x+1)|+C$

e) $\frac{1}{2}\ln\left|\frac{2x+1}{2x-1}\right|+C$

5 pts 6. Find

$$\lim_{x\to 0}\frac{e^x-x-1}{\sin^2 x}.$$

a) 0

b) 1

c) $\frac{1}{2}$

d) 2

e) ∞

5 pts

7. It is known that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} = \frac{\pi^3}{32}$. Find the smallest number of terms, n, of this series needed to be certain that the partial sum $S_n = \displaystyle\sum_{k=1}^{n} \frac{(-1)^{k+1}}{k^3}$ is within .001 of $\dfrac{\pi^3}{32}$.

a) 5

b) 9

c) 12

d) 14

e) 20

5 pts

8. If $\displaystyle\lim_{n\to\infty} a_n = 1$, then $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+a_n}$ must

a) converge to 0

b) converge to $\dfrac{1}{2}$

c) converge to 1

d) converge to e

e) diverge

5 pts 9. Find the sum of the series

$$\sum_{n=2}^{\infty} \left(-\frac{1}{2}\right)^n$$

a) $\frac{1}{2}$

b) -1

c) $\frac{2}{3}$

d) $-\frac{1}{3}$

e) $\frac{1}{6}$

5 pts 10. A radioactive substance is known to have a half-life of 8 hours and an initial mass of 600 grams. Find the mass of the substance after 4 hours.

a) $300\sqrt{2}$ grams

b) $600\sqrt{2}$ grams

c) $\frac{8 \ln 50}{\ln 2}$ grams

d) 450 grams

e) $150\sqrt{2}$ grams

5 pts 11. Find the length of the curve given by the parametrization

$$x = \frac{t^2}{2}, \quad y = \frac{t^3}{3}, \quad 0 \le t \le 1$$

a) $\sqrt{2}$

b) $\dfrac{2^{3/2} - 1}{3}$

c) $\dfrac{2^{3/2}}{3}$

d) $\dfrac{4\sqrt{2} - 2}{3}$

e) $\dfrac{5}{6}$

5 pts 12. Determine which one of the following is the graph of $r = 1 + \sin\theta$ in polar coordinates.

5 pts 13. Given the curve $x = 2\sin t$, $y = 3\cos t$, find the slope of the tangent line to the curve at $t = \frac{\pi}{4}$.

a) $-\frac{1}{3}$

b) $-\frac{3}{2}$

c) 1

d) ∞

e) $-\frac{2}{3}$

5 pts 14. Express the polar equation $r = 4\tan\theta\sec\theta$ in Cartesian coordinates.

a) $x^2 + y^2 = 4x$

b) $x^2 + y^2 = \frac{4y}{x^2}$

c) $\sqrt{x^2 + y^2} = \frac{4y}{x^2}$

d) $y^2 = 4x$

e) $x^2 = 4y$

9 pts 15. (NO PARTIAL CREDIT) Determine if each sequence below converges or diverges. Circle your answer. If the sequence converges you must give the limit to receive credit for this answer.

a) $\left\{ \dfrac{\ln(3n)}{\ln(2n+1)} \right\}$ (A) diverges (B) converges to ________

b) $\left\{ \left(\dfrac{n-5}{n} \right)^n \right\}$ (A) diverges (B) converges to ________

c) $\{n^{1/n}\}$ (A) diverges (B) converges to ________

9 pts 16. Determine if each of the series below is absolutely convergent, conditionally convergent or divergent. Circle your answer. (**AC:** Absolutely Convergent; **CC:** Conditionally Convergent; **D:** Divergent.)

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ **AC** **CC** **D**

b) $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$ **AC** **CC** **D**

c) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2+1}$ **AC** **CC** **D**

10 pts 17. (PARTIAL CREDIT PROBLEM) Evaluate

$$\int_4^9 \frac{dx}{x - \sqrt{x}}\,.$$

10 pts 18. (PARTIAL CREDIT PROBLEM) Evaluate

$$\int_0^\infty xe^{-x}\, dx\,.$$

14 pts 19. (PARTIAL CREDIT PROBLEM) You may write series using either summation $\left(\sum\right)$ notation, or by writing out at least the first **four nonzero** terms.

a) (4 pts.) State the Maclaurin series for $\sin x$.

b) (5 pts.) Find the Maclaurin series for $3\sin(x^2)$ by using your answer from part (a).

c) (5 pts.) Find the Maclaurin series representation of

$$\int 3\sin(x^2)\ dx$$

by using your answer from part b).

14 pts 20. (PARTIAL CREDIT PROBLEM) Find the radius R and interval I of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^n}{\sqrt{2n}}.$$

Show all work clearly and put your answers into the blanks provided.

a) (7 pts.) $\mathbf{R} =$ ________________ , (7 pts.) $\mathbf{I} =$ ________________

14 pts 21. (PARTIAL CREDIT PROBLEM)

a) (8 pts.) Find the polar coordinates of all intersection points of the polar curves $r^2 = \cos(2\theta)$ and $r = \frac{\sqrt{2}}{2}$. Label them in the picture below.

b) (6 pts.) Set up an integral or integrals (**but DO NOT evaluate the integral(s)**) which measure the area of the shaded region shown above.