

MATH 141

NAME _____

FINAL EXAMINATION

STUDENT NUMBER _____

MAY 4, 2004

INSTRUCTOR _____

TEST FORM A

SECTION NUMBER _____

This examination consists of 20 problems. The first 14 are multiple choice questions, the next two are short answer questions and the remaining 4 are partial credit problems. The point value for each question appears to the right of the question number. There are 150 total points.

Please record your answers to the multiple choice questions by circling the corresponding letter. Present your work clearly for the partial credit problems. **No credit will be given for unsupported answers.**

THE USE OF CALCULATORS, BOOKS, NOTES ETC.

DURING THIS EXAMINATION IS PROHIBITED.

Do not write in the blanks below.

(MC) _____(84)

15. _____(12)

16. _____(9)

17. _____(9)

18. _____(10)

19. _____(15)

20. _____(11)

TOTAL _____

1. (6 pts.) Differentiate $2^{\sin x}$.

a) $\cos x \cdot \sin x \cdot 2^{\sin x}$

b) $\cos x \cdot 2^{\sin x}$

c) $\frac{2^{\sin x}}{\cos x}$

d) $\ln 2 \cdot \cos x \cdot 2^{\sin x}$

e) $\frac{2^{\sin x}}{\ln 2}$

2. (6 pts.) Evaluate $\log_2 \left(\frac{\ln(e^2)}{\log_3(81)} \right)$.

a) -3

b) -2

c) -1

d) 0

e) 1

3. (6 pts.) Evaluate $\lim_{x \rightarrow 0} \frac{x - x \cos x}{x^2 + \tan^{-1} x}$.

- a) 0
- b) 1
- c) ∞
- d) $-\infty$
- e) $\frac{\pi}{2}$

4. (6 pts.) Evaluate $\int_1^e x \ln x \, dx$.

- a) $\frac{x^2}{2} \ln x - \left(\frac{e^2}{4} - \frac{1}{4}\right)$
- b) $\frac{e^2}{4} + \frac{1}{4}$
- c) $2 \ln(e^2 + 1)$
- d) $\frac{1}{4} - \frac{\sqrt{e}}{2}$
- e) $\ln 2 \left(\frac{e^2 + 1}{3}\right)$

5. (6 pts.) Evaluate $\int_0^{\pi/2} \sin^3 x \cos^5 x \, dx$.

a) $\frac{1}{24}$

b) $\frac{3}{24}$

c) $-\frac{3}{24}$

d) $-\frac{1}{12}$

e) $\frac{1}{12}$

6. (6 pts.) If $f(x) = x^3 + 2x^2 - 1$ then $f(x)$ is one-to-one when $x > 0$. Let $g(x) = f^{-1}(x)$. What is $g'(2)$?

a) $\frac{1}{2}$

b) $\frac{1}{5}$

c) $\frac{1}{7}$

d) $\frac{1}{17}$

e) $\frac{1}{20}$

7. (6 pts.) Evaluate $\int \frac{2x - 13}{(2x + 1)(x - 3)} dx$.

a) $\ln |(2x + 1)(x - 3)^2| + C$

b) $\ln |2x^2 - 5x - 3| + C$

c) $\ln \left(\frac{(2x + 1)^2}{|x - 3|} \right) + C$

d) $\ln \left(\frac{(2x + 1)^4}{|x - 3|} \right) + C$

e) $\ln \left| \frac{x - 3}{2x + 1} \right| + C$

8. (6 pts.) Evaluate $\int_{-\infty}^{\infty} \frac{1}{1 + 4x^2} dx$.

a) 0

b) $\pi/4$

c) $\pi/2$

d) π

e) The integral diverges.

9. (6 pts.) If $\sum_{n=4}^{\infty} a_n = 10$, then what is $\lim_{n \rightarrow \infty} \frac{\sin(a_n)}{a_n + 1}$?

- a) 0
- b) 1
- c) $\frac{\sin(10)}{11}$
- d) $\frac{\sin(4)}{5}$
- e) The sequence diverges.

10. (6 pts.) Find the sum of the series $\sum_{n=0}^{\infty} (-5) \frac{4^{n+1}}{3^{2n}}$.

- a) $-\frac{180}{13}$
- b) -36
- c) -9
- d) -1
- e) The series diverges.

11. (6 pts.) Find the Maclaurin series for $\sin(3x^2)$.

a)
$$\sum_{n=0}^{\infty} (-1)^n 3^{4n+2} \frac{x^{2n+1}}{(2n+1)!} = 3^2 x - \frac{3^6}{3!} x^3 + \frac{3^{10}}{5!} x^5 - \frac{3^{14}}{7!} x^7 + \dots$$

b)
$$\sum_{n=0}^{\infty} (-1)^n \frac{3}{(2n)!} x^{4n} = 3 - \frac{3}{2!} x^4 + \frac{3}{4!} x^8 - \frac{3}{6!} x^{12} + \dots$$

c)
$$\sum_{n=0}^{\infty} (-1)^n \frac{3}{(2n+1)!} x^{4n+2} = 3x^2 - \frac{3}{3!} x^6 + \frac{3}{5!} x^{10} - \frac{3}{7!} x^{14} + \dots$$

d)
$$\sum_{n=0}^{\infty} (-1)^n 3^{2n} \frac{x^{4n}}{(2n)!} = 1 - 3^2 \frac{x^4}{2!} + 3^4 \frac{x^8}{4!} - 3^6 \frac{x^{12}}{6!} + \dots$$

e)
$$\sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1}}{(2n+1)!} x^{4n+2} = 3x^2 - \frac{3^3}{3!} x^6 + \frac{3^5}{5!} x^{10} - \frac{3^7}{7!} x^{14} + \dots$$

12. (6 pts.) If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 4, then what is the radius of convergence of $\sum_{n=0}^{\infty} c_n x^{2n}$?

- a) 1
- b) 2
- c) 4
- d) 8
- e) 16

13. (6 pts.) Find the power series for $\frac{1}{3+x}$ centered at 0.

a) $\sum_{n=0}^{\infty} (3+x)^n$

b) $\sum_{n=0}^{\infty} 3^n x^n$

c) $\sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$

d) $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{3^{n+1}}$

e) $\sum_{n=0}^{\infty} -\frac{1}{9} x^n$

14. (6 pts.) What is the slope of the tangent line to the curve defined by $x = t^2 - 1$ and $y = 2e^t$ at $t = 2$?

a) $\frac{e^2}{2}$

b) e^2

c) $\frac{e^2}{4}$

d) $2e^2$

e) e^{-2}

15. (12 pts.) Determine if each of the series below is absolutely convergent, conditionally convergent or divergent. Circle your answer (**AC**: Absolutely Convergent; **C**: Conditionally convergent; **D** Divergent.) NOTE: You do not need to show your work.

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n-4}$ **AC** **CC** **D**

b) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{e^n + n}$ **AC** **CC** **D**

c) $\sum_{n=1}^{\infty} \frac{3 - \cos n}{n^{2/3} - 2}$ **AC** **CC** **D**

d) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2}$ **AC** **CC** **D**

16. (9 pts.) Determine whether the following sequences converge or diverge. For credit, circle your answer choice and find the limit of each convergent sequence. NOTE: You do not need to show your work.

a) $\left\{ \frac{n^2}{n+1} - \frac{n^2}{n+3} \right\}$ Diverges Converges to _____

b) $\left\{ n \cdot \tan\left(\frac{1}{n}\right) \right\}$ Diverges Converges to _____

c) $\left\{ \left(2 + \frac{1}{n}\right)^n \right\}$ Diverges Converges to _____

17. (9 pts.) (PARTIAL CREDIT PROBLEM) Evaluate $\int \frac{x-1}{x+\sqrt{x}} dx$. For credit you must show all work that leads to your answer.

18. (10 pts.) (PARTIAL CREDIT PROBLEM) Find the radius \mathbf{R} of convergence and the interval \mathbf{I} of convergence for the series $\sum_{n=1}^{\infty} (-1)^n \frac{(2x+3)^n}{n9^n}$. Record your answers in the spaces provided at the bottom of the page.

$\mathbf{R} =$ _____, $\mathbf{I} =$ _____

19. (15 pts.) (PARTIAL CREDIT PROBLEM)

a) (5 pts.) Give the general formula for the Taylor series of the function $f(x)$ centered at a .

b) (5 pts.) Find $T_3(x)$ (the third degree Taylor polynomial) for $f(x) = e^x$ centered at $a = 2$.

c) (5 pts.) Estimate the accuracy of the approximation $e^x \approx T_3(x)$ when $1.5 \leq x \leq 2.5$.

20. (11 pts.) (PARTIAL CREDIT PROBLEM)

a) (6 pts.) Sketch the graph of the polar curve $r = 4 - 4 \sin \theta$.

b) (5 pts.) **Set up, but do not evaluate** the integral for the area enclosed by this curve.