1. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then which of the following statements is true?

   a) A and B are both in reduced echelon form.

   b) A is in reduced echelon form, but B is not.

   c) B is in reduced echelon form, but A is not.

   d) Neither A nor B is in reduced echelon form.

2. If the augmented matrix of a linear system is give by $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then what is the general solution of the system?

   a) $x_1 = x_3$, $x_2 = 1 - x_3$, x_3 is free

   b) $x_1 = 4 - 3x_3$, $x_2 = 2 - 2x_3$, x_3 is free

   c) $x_1 = -2 + 3x_3$, $x_2 = 4 - 4x_3$, x_3 is free

   d) $x_1 = x_3$, $x_2 = 2 - 2x_3$, x_3 is free

3. Which of the following vectors is in the span of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$?

   a) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

   b) $\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

   c) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

   d) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

4. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ 3x_1 \\ x_1 + 2x_2 \end{bmatrix}$, then what is the standard matrix of T ?

   a) $\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix}$

   b) $\begin{bmatrix} -1 & 2 \\ 3 & 0 \\ 2 & 1 \end{bmatrix}$

   c) $\begin{bmatrix} -1 & 2 \\ 3 & 1 \\ 2 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$

5. If T is the linear transformation whose standard matrix is $A = \begin{bmatrix} 1 & -4 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, then which of the following statements is true ?

   a) T is onto but not one-to-one.

   b) T is both onto and one-to-one.

   c) T is one-to-one but not onto.

   d) T is neither one-to-one nor onto.

6. If $A = \begin{bmatrix} 0 & 2 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, then what is AB?

   a) $\begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 2 \end{bmatrix}$

   b) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

   c) $\begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} -2 & -2 & 3 \\ 2 & 3 & 3 \end{bmatrix}$

7. If

$$A = \begin{bmatrix} 1 & -2 & 0 & -1 \\ -1 & 2 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   then what is the rank of A?

   a) 1

   b) 2

   c) 3

   d) 4

8. If $\mathbf{u} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$, then what is $\mathbf{u}^T\mathbf{u}$?

a) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$

b) 1

c) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

9. If $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix}$, then evaluate $\det(A)$.

a) 4

b) -4

c) 2

d) -2

10. If A, B and C are $n \times n$ matrices and $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, then find $\det(A^2BC^{-1})$.

a) 1

b) $\frac{123}{5}$

c) 60

d) -60

11. If $A = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$, then find $\det(4A)$.

a) 1

b) 4

c) 16

d) 32

12. If $A = \begin{bmatrix} -3 & 0 & 1 & 0 \\ 4 & 2/3 & 0 & -3 \\ -2 & 5 & -1 & 0 \\ -7 & 1/3 & 8 & 3 \end{bmatrix}$, then evaluate $\det(A)$

a) 15

b) 15

c) 6

d) -6

13. If $A = \begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix}$, then what is A^{-1}?

a) $\frac{1}{6}\begin{bmatrix} -2 & 4 \\ 4 & -5 \end{bmatrix}$

b) $\frac{1}{6}\begin{bmatrix} 2 & -4 \\ -4 & 5 \end{bmatrix}$

c) $\frac{-1}{6}\begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix}$

d) $\frac{1}{6}\begin{bmatrix} 5 & -4 \\ -4 & 2 \end{bmatrix}$

14. If $A = \begin{bmatrix} \lambda - 3 & 3 & 2 \\ 0 & \lambda - 2 & 1 \\ 0 & 0 & \lambda \end{bmatrix}$, then find all values of λ for which $\det(A) = 0$

a) $\lambda = 3$ and $\lambda = 2$

b) $\lambda = 3, \lambda = 2$ and $\lambda = 0$

c) $\lambda = 3, \lambda = 2$ and $\lambda = 1$

d) $\lambda = 0$

15. If $A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is an eigenvector for A, then what is the corresponding eigenvalue?

a) $\lambda = 1$

b) $\lambda = 2$

c) $\lambda = \frac{1}{2}$

d) $\lambda = -1$

16. What is the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$?

a) $\lambda^2 + 5\lambda + 5 = 0$

b) $\lambda^2 - 6\lambda + 7 = 0$

c) $\lambda^2 - 5\lambda + 7 = 0$

d) $\lambda^2 - 5\lambda + 6 = 0$

17. What are the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$?

a) $\lambda = 0$ with the multiplicity 3

b) $\lambda = 1, \lambda = 2, \lambda = 3$

c) $\lambda = 0, \lambda = 1, \lambda = 3$

d) $\lambda = -1, \lambda = 1, \lambda = 2$

18. If $A = \begin{bmatrix} 1 & 5 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, then what is the dimension of the eigenspace corresponding to the eigenvalue $\lambda = 3$?

a) 0

b) 1

c) 2

d) 3

19. If $A = \begin{bmatrix} 2 & 2\sqrt{3} \\ 2\sqrt{3} & 3 \end{bmatrix}$ and $A = PDP^{-1}$ is the diagonalization of A, then find D.

a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 6 & 0 \\ 0 & -1 \end{bmatrix}$

c) $\begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$

20. If $\mathbf{u} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ then evaluate$(u \cdot v)w + (w \cdot u)(-v)$.

a) 0

b) $\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} 3 \\ 3 \\ 0 \\ -6 \end{bmatrix}$

d) -10

21. For which value of h are the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} h \\ 0 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix}$ orthogonal?

a) $h = 0$

b) $h = 2$

c) $h = -2$

d) $h = 4$

22. If $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then what is the projection of $\mathbf{u}$ onto $W = \text{Span}\{\mathbf{w}\}$?

a) 0

b) $\begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$

c) $\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$

d) $\begin{bmatrix} -1/3 \\ 1/3 \\ -1/3 \end{bmatrix}$

23. Find the matrix of the quadratic form $3x_1^2 + 6x_1x_2 - 2x_2^2$.

a) $\begin{bmatrix} 3 & 6 \\ -2 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 3 \\ 3 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 3 & -2 \\ 6 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$

24. Compute the value of the quadratic form $Q(\mathbf{x})$ with the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ for $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

a) 2

b) 1

c) 0

d) -2

25. Which one of the following matrices is symmetric ?

a) $\begin{bmatrix} 1 & 5 & 2 \\ 4 & 7 & 0 \\ -1 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 5 & 2 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 7 & 0 \\ -1 & 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 5 & 2 \\ 5 & 7 & 0 \\ 2 & 0 & 1 \end{bmatrix}$