1. The linear system

$$x_1 - 2x_2 = 6$$
$$2x_1 - x_2 + 3x_3 = 4$$
$$x_2 + x_3 = 1$$

has a solution set:

a) $x_1 = 8 - 2x_3$, $x_2 = 1 - x_3$, x_3 free

b) $4x_1 = 6$, $x_2 = 0$, $x_3 = 1$

c) $x_1 = 6 + 2x_2$, x_2, x_3 free

d) It is inconsistent

2. Which of the following sets is a subspace of $\mathbb{R}^3$?

$$S_1 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 x_2 = 0 \right\} \qquad S_2 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 = 0 \right\}$$

$$S_3 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : 3x_1 - x_2 + 5x_3 = 6 \right\} \qquad S_4 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + 2x_2 + 3x_3 = 0 \right\}$$

a) S_1 and S_2

b) S_1, S_2 and S_4

c) S_2 and S_4

d) S_2, S_3 and S_4

3. Let $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$. Find $\det(2A^9 B^{-1})$.

a) -1

b) -2

c) 1

d) 2

4. Find the necessary condition on $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ for the system $A\mathbf{x} = \mathbf{b}$ to be consistent, where $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 1 \\ 6 & 0 & 1 \end{bmatrix}$.

a) $3b_1 - b_2 = 0$

b) $2b_1 + b_2 + 2b_3 = 0$

c) $2b_1 + b_2 - b_3 = 0$

d) It is consistent for every b

5. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 0 & 2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$. Which of the following is true?

a) $A\mathbf{x} = \mathbf{b}$ has a unique solution.

b) $A\mathbf{x} = \mathbf{b}$ has exactly three solutions.

c) $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

d) $A\mathbf{x} = \mathbf{b}$ is inconsistent.

6. Reflection in the x_1-axis is a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$. What is the standard matrix for this transformation?

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

d) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

7. Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ 3 & 7 & -2 \end{bmatrix}$$

The second column of A^{-1} is

a) $\begin{bmatrix} -2 \\ 0 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

d) A is not invertible

8. A is $n \times n$ matrix and its rank is $n-2$. The dimension of its nullspace is

a) 1

b) 2

c) n

d) $n - 1$

9. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 4 & 9 & 4 \end{bmatrix}$. Find $\det(A)$.

   a) 0

   b) 10

   c) 20

   d) 12

10. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 0 & 0 \\ -6 & 0 & 2 \end{bmatrix}$ is

   a) $(3-\lambda)(5-\lambda)(-6-\lambda)$

   b) $-\lambda^3 + 5\lambda^2 - 6\lambda$

   c) $\lambda^2 - 5\lambda + 6$

   d) 0

11. Let $A = \begin{bmatrix} 1 & -1 & -3 \\ -2 & 1 & 4 \\ 0 & 3 & 2 \end{bmatrix}$. Then the vector $\mathbf{x} = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}$ is the eigenvector for A with the eigenvalue

   a) 1

   b) -1

   c) -2

   d) 2

12. Let $A = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}$. What is the matrix P in the diagonalization $A = PDP^{-1}$?

   a) $\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

13. Let $\mathbf{u}_1 = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 3/\sqrt{14} \\ -2/\sqrt{14} \\ 1/\sqrt{14} \end{bmatrix}$. Which of the following is true ?

   a) $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthonormal set of vectors.

   b) $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal but not orthonormal set of vectors.

   c) $\{\mathbf{u}_1, \mathbf{u}_2\}$ are linearly dependent.

   d) Span $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $\mathbb{R}^3$.

14. Let $\mathbf{x} = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$. Find the orthogonal projection of $\mathbf{x}$ onto the plane W spanned by the orthogonal vectors $\{\mathbf{u}_1, \mathbf{u}_2\}$.

   a) $\begin{bmatrix} -5/2 \\ 5/2 \\ 5 \end{bmatrix}$

   b) $\begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$

   c) $\begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$

   d) $\begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$

15. Which of the following matrices is symmetric?

   a) $\begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 0 & 2 \\ -5 & 2 & 0 \end{bmatrix}$

   d) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

16. Let $\mathbf{u}_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 8 \\ -2 \\ -1 \end{bmatrix}$. Use Gram-Schmidt process to find an orthogonal basis for the subspace W=Span $\{\mathbf{u}_1, \mathbf{u}_2\}$.

   a) $\left\{ \begin{bmatrix} -1/\sqrt{5} \\ 0 \\ 2/\sqrt{5} \end{bmatrix}, \begin{bmatrix} 8/\sqrt{69} \\ -2/\sqrt{69} \\ -1/\sqrt{69} \end{bmatrix} \right\}$

   b) $\left\{ \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right\}$

   c) $\left\{ \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix} \right\}$

   d) $\left\{ \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 10 \\ -2 \\ -5 \end{bmatrix} \right\}$

17. For $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ find an **orthogonal matrix** P such that $A = P \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} P^{-1}$.

   a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

   b) $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

18. For $\mathbf{x}$ in $\mathbb{R}^3$, let $Q(\mathbf{x}) = 2x_2^2 - 6x_3^2 + x_1x_2 - 4x_2x_3$. The matrix of the quadratic form $Q(\mathbf{x})$ is

   a) $\begin{bmatrix} 2 & 1/2 & -2 \\ 1/2 & -6 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

   b) $\begin{bmatrix} 0 & 1/2 & 0 \\ 1/2 & 2 & -2 \\ 0 & -2 & -6 \end{bmatrix}$

   c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & -4 \\ 0 & 0 & -6 \end{bmatrix}$

   d) $\begin{bmatrix} 2 & 1 & -4 \\ 1 & -6 & 0 \\ -4 & 0 & 0 \end{bmatrix}$

19. Let A be a square matrix. Which of the following statements is always TRUE?

   a) $A + A^T$ is always invertible.

   b) $A - A^T$ is always the zero matrix.

   c) $\det(A - A^T)$ is always zero.

   d) $A + A^T$ is always diagonalizable.

20. Let $\mathbf{x} = P\mathbf{y}$ be a change of variables that transforms the quadratic form $Q = x_1^2 + 4x_2^2 - 4x_1x_2$ into a quadratic form with no cross-product terms. Then the new quadratic form is

   a) $y_1^2 + 4y_2^2 - 4y_1y_2$

   b) $y_1^2 + 4y_2^2$

   c) $5y_2^2$

   d) $y_1^2 + 5y_2^2$

21. Find a basis for the subspace of $\mathbb{R}^3$ which consists of all vectors that are orthogonal to both the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

   a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

   b) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

   c) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

   d) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

22. Let $A = \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}$, and let $A = PDP^{-1}$ be a diagonalization of A with $P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Find A^{2001}.

   a) $A^{2001} = \begin{bmatrix} (-3)^{2001} & 4^{2001} \\ (-2)^{2001} & 3^{2001} \end{bmatrix}$

   b) $A^{2001} = \begin{bmatrix} (-1)^{2001} & 0 \\ 0 & 1^{2001} \end{bmatrix}$

   c) $A^{2001} = A$

   d) $A^{2001} = I$

23. If the eigenvalues of a 3×3 matrix A are $1, 1, -1$ then which of the following is ALWAYS TRUE?

   a) A is invertible.

   b) A is not invertible.

   c) A is diagonalizable.

   d) A is not diagonalizable.

24. Calculate $\mathbf{u} \cdot (2\mathbf{v} + \mathbf{w})$ where $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$.

   a) 0

   b) 2

   c) 3

   d) 4

25. Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$. What is the dimension of the eigenspace for the eigenvalue $\lambda = -1$.

a) 4

b) 3

c) 2

d) 1