MATH 230

Exam 1

February 14, 2♡♡5

NAME ______________________________

STUDENT NUMBER ______________________

INSTRUCTOR __________________________

SECTION NUMBER ______________________

This exam contains 10 free-response questions on 10 pages (including this title page). This exam is worth a total of 100 points. To receive full credit for a problem all work must be shown. When in doubt, fill in the details. **No notes, books or calculators may be used during this exam.**

Please Box Your Final Answers

(when possible).

10 pts 1. Determine the equation of the plane containing the two parallel lines

$$L_1: \qquad x = -6t, \qquad y = 1 + 9t, \qquad z = -3t$$

and

$$L_2: \qquad x = 1 + 2s, \qquad y = 4 - 3s, \qquad z = s.$$

Write the equation in standard form: $ax + by + cz + d = 0$.

10 pts 2.

6 pts a) Convert the given set of spherical coordinates to cylindrical coordinates and to rectangular coordinates.

$$(\rho, \theta, \phi) = (2, \frac{\pi}{4}, \frac{\pi}{6})$$

4 pts b) Write the given equation in rectangular coordinates:

$$\rho^2(2\sin^2\phi - \cos^2\phi) = 1$$

12 pts 3. Consider the curve

$$\mathbf{r}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j} + \sqrt{2}t\mathbf{k} \qquad 0 \leqslant t \leqslant 2$$

8 pts a) Find the unit tangent and unit normal vectors of this curve, both as functions of time.

4 pts b) Find the curvature of this curve as a function of time.

8 pts 4. Find the arclength of the following curve for $1 \leqslant t \leqslant 3$.

$$\mathbf{r}(t) = \frac{t^2}{2}\mathbf{i} + \frac{2\sqrt{2}}{3}t^{3/2}\mathbf{j} + t\mathbf{k}$$

10 pts 5. Consider a particle whose acceleration is given by

$$\mathbf{a}(t) = -\cos t\mathbf{i} - \sin t\mathbf{j} + (2t-1)\mathbf{k}$$

With initial velocity $\mathbf{v}(0) = \langle 0, 1, 2\rangle$ and initial position $\mathbf{r}(0) = \langle 1, 0, 0\rangle$.

4 pts a) Find the velocity of the particle as a function of time.

4 pts b) Find the position of the particle as a function of time.

2 pts c) Find the speed at time $t = 1$.

10 pts 6. Determine whether the following limits exist. Prove your claim.

5 pts a)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x + y},$$

5 pts b)

$$\lim_{(x,y)\to(0,0)} \frac{3x^3y}{x^4 + 2y^4}.$$

10 pts 7. Find all the second partial derivatives of

$$f(x, y) = e^{-xy} \cos x.$$

10 pts 8. Find the parametric equations of the line tangent to the curve of intersection of the two surfaces, $z = x^2 + 3y^2$ and $x + y + z = 4$ at the point $(1, -1, 4)$.

10 pts 9. For each problem circle T if the statement is true and F if the statement is false.

a) **T** **F** A direction and a point always determine a unique line.

b) **T** **F** Three distinct points always determine a unique plane.

c) **T** **F** Two planes perpendicular to a third plane are parallel to each other.

d) **T** **F** Hyperbolas have negative curvature.

e) **T** **F** The vectors $\langle t, t\sin^2 t, -\cos t\rangle$ and $\langle -1/t, 1/t, -\cos t\rangle$ are perpendicular for $t \neq 0$.

f) **T** **F** $(2-x)^2 + (-3-y)^2 + (5-z)^2 = 6$ is a sphere of radius $\sqrt{6}$ and center $(-2, 3, -5)$.

g) **T** **F** $\mathbf{r}(t) \times |\mathbf{r}'(t) \times \mathbf{r}''(t)|$ is a vector.

h) **T** **F** For all vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$: $|\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a})| = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.

i) **T** **F** If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ then $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$.

j) **T** **F** f_{xy} and $\dfrac{\partial^2 f}{\partial y \partial x}$ both mean to differentiate f first with respect to y and then with respect to x.

10 pts 10. Let a particle's path be given by

$$\mathbf{r}(t) = \ln t\mathbf{i} + t^2\mathbf{j} + t^2\mathbf{k}.$$

Calculate the vector projection

$$\text{proj}_{\mathbf{v}(t)}\mathbf{a}(t)$$

when $t = 1$.