MATH 230
Spring 2005
Exam 2
Spring, 2005

NAME :_____

This exam contains 10 free-response questions on 11 pages (including this title page). This exam is worth a total of 100 points. To receive full credit for a problem all work must be shown. When in doubt, fill in the details. **No notes, books or calculators may be used during this exam**.

Please Box Your Final Answers

(when possible).

1:_____
2:_____
3:_____
4:_____
5:_____
6:_____
7:_____
8:_____
9:_____
10:_____
Total:_____

1. (12 points) Suppose you are climbing a hill whose shape is given by the equation $z = 7 - 3x^2 - y^2$, and you are standing at a point with coordinates $(0, 1, 6)$.
   (a) (6 points) What is the slope of the trail in the direction of $< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$?
   (b) (3 points) In which direction should you walk to stay at constant elevation?
   (c) (3 points) What is the slope of the trail with the steepest ascent?

2. (10 points) Find and classify all the critical points of the following function.

$$f(x,y) = xy^2 + \frac{1}{4}x^2 - 4x$$

3. (10 points) Find the maximum and minimum values of the function

$$f(x, y) = x^2 y$$

subject to the constraint that $x^2 + y^2 = 1$.

4. (10 points) Compute the integral by interpreting it as a volume: (Do NOT use the iterated integration.)

$$\iint_D \left(2 - \sqrt{4 - x^2 - y^2}\right)\, dA$$

where $D = \{(x, y) \mid x^2 + y^2 \leq 4\}$.

5. (10 points) Evaluate the integral by changing the order of integration:

$$\int_0^2 \int_{y^2}^4 y e^{x^2} \, dx dy$$

6. (10 points) Evaluate the following double integral, where the region R on xy-plane is bounded by $y = x,\ y = 0, x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

$$\iint_R \frac{1}{(x^2+y^2)^{\frac{3}{2}}}\, dydx$$

7. (10 points) Find the area of the surface which is the part of the cone $z = 4 - \sqrt{x^2 + y^2}$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

8. (8 points) Find the average value of the function $f(x, y, z) = y \sin x + z$ over the region $E = \{(x, y, z) \mid 0 \leq x \leq 2\pi,\ 0 \leq y \leq 4,\ 0 \leq z \leq 1\}$.

9. (6 points) Consider the region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and the cone $z = \sqrt{x^2 + y^2}$, and below by the plane $z = 0$. Write down an integral (in spherical coordinates) that will give the volume of this region.
DO NOT solve this integral.

10. (14 points) Consider the following coordinate transform.

$$u = x + y \qquad v = x - y$$

(a) (3 points) Invert this transform, i.e. find functions $x(u, v)$ and $y(u, v)$.

(b) (4 points) Find the Jacobian of the transform from part (a).

(c) (7 points) Using the above two find

$$\iint_D (x^2 - y^2)\, dA$$

Where D is bounded by $1 \leqslant x - y \leqslant 2$ and $0 \leqslant x + y \leqslant 3$.