Problem	Score
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Name: ____________

SID: ____________

Section: ____________

Instructor: ____________

MATH 230 FALL 2004
FINAL EXAM
DECEMBER 13, 2004 12:20-2:10 PM

INSTRUCTIONS

- There are **12** problems on this exam for a total of **150** points. Some problems have multiple parts.
- PLEASE, SHOW YOUR WORK. ANSWERS WITHOUT SUPPORTING WORK WILL BE GIVEN NO CREDIT.
- Be sure your answers are legible and complete.
- You **may not** use CALCULATORS, BOOKS, or PERSONAL NOTES.
- **Do not** write on the line marked SCORE at the bottom of each page.
- Cellular phones must be turned off at the beginning of the exam.

1. (9 points) Suppose the equation a) or b) or c) below holds for two vectors $\vec{u}$ and $\vec{v}$, that are not the zero vector. Determine for each of a), b), c), whether the two vectors are parallel, perpendicular or whether it cannot be decided:

   (a) (3 points) $\vec{u} \cdot \vec{v} = 0$ $\|$ $\perp$ undecided

   (b) (3 points) $\vec{u} \times \vec{v} = \vec{0}$ $\|$ $\perp$ undecided

   (c) (3 points) $(\vec{u} \times \vec{v}) \cdot (\vec{v} \times \vec{u}) = 0$ $\|$ $\perp$ undecided

2. (12 points) There is a mountain represented by the graph of the function $z = \sin(xy) + e^y$.

   (a) (6 points) To climb up the FASTEST, in which direction should you walk at the location $(2, 0, 1)$?

   (b) (6 points) To stay at the SAME elevation, in which directions should you walk at the location $(2, 0, 1)$?

3. (10 points) Find parametric equations of the sphere centered at the point $(1, 1, 0)$ with radius 1.

   Hint: Think of spherical coordinates.

4. (a) (6 points) Find all critical points of the function $f(x, y) = x^2 - 3y + y^3$.

   (b) (7 points) Determine if they are points of local maximum, local minimum, or saddle points.

5. (14 points) Convert the triple integral

$$\iiint_E x\,dV,$$

where E is the solid region in space inside the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 4$, into:

(a) (7 points) a triple integral in <u>cylindrical</u> coordinates (DO NOT EVALUATE THE INTEGRAL);

(b) (7 points) a triple integral in <u>spherical</u> coordinates (DO NOT EVALUATE THE INTEGRAL).

6. (10 points) Find the linearization (or tangent plane approximation) of the function $z = f(x, y) = e^{x^2+y^2}$ at the point $P(1, 1, e^2)$.

7. (14 points) Evaluate the surface integral

$$\iint_S y\, dS,$$

where S is the part of the plane $y+3-z=0$ that lies inside the cylinder $x^2+y^2=1$.

8. (13 points) A driver is supposed to drive slowly at a curve to stay on the road. Car A is driving along a curve with curvature 3 at the time $t = 0$. At the same time, Car B is driving along the curve

$$\vec{r}(t) =< 2\cos t, 2\sin t, t > .$$

At time $t = 0$, which of the two cars is supposed to drive the MOST SLOWLY?

SCORE: ______________

9. (15 points) Let $\vec{F}(x,y) = (2x + \sin y)\,\vec{i} + x\cos y\,\vec{j}$.

   (a) (5 points) Show $\vec{F}$ is conservative.

   (b) (10 points) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the arc of parabola $y = 1 - x^2$ in the first quadrant from $(0,1)$ to $(1,0)$.

10. (14 points) Use the Divergence Theorem to find the INWARD flux of the vector field
$$\vec{F}(x, y, z) = yz\,\vec{i} + y\,\vec{j} + z\,\vec{k}$$
across the sphere $x^2 + y^2 + z^2 = 4$.

SCORE: ______________

11. (16 points) Consider the vector field $\vec{F}(x, y, z) =< -y, x, x - 2y >$.

(a) (6 points) Find $\text{curl}\,\vec{F}$.

(b) (10 points) Use Stokes' theorem to compute the UPWARD flux of $\text{curl}\,\vec{F}$ across the paraboloid $z = 1 - x^2 - y^2$ with $z \geq 0$.

12. (10 points) Write the equation for the tangent plane to the parametric surface represented by

$$\vec{r}(u,v) =< u, u\,\cos v, u\,\sin v >, \qquad u \geq 0, \quad 0 \leq v \leq 2\pi,$$

at the point $P(1,0,1)$.