MATH 230
Spring 2005
Final Exam
May 3, 2005

NAME :____________________
INSTRUCTOR:____________________

This exam contains 13 free-response questions on 14 pages (including this title page). This exam is worth a total of 150 points. To receive full credit for a problem all work must be shown. When in doubt, fill in the details. **No notes, books or calculators may be used during this exam.**

Please Box Your Final Answers

(when possible).

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1. (10 points) Suppose the acceleration of a particle in space is given by

$$\mathbf{a}(t) = \langle \sin t, \cos t, 6t \rangle .$$

Suppose the particle starts at rest, i.e., $\mathbf{v}(0) = 0$, with initial position given by $\langle 1, 2, 3 \rangle$.

(a) (4 points) Find the velocity $\mathbf{v}(t)$ at any time t.

(b) (4 points) Find the position $\mathbf{r}(t)$ at any time t.

(c) (2 points) Find the speed of the particle after π seconds.

2. (10 points) The curve C consists of two smooth pieces: the first, C_1, is a straight line segment connecting the points $(3, -1)$ and $(1, 1)$ and the second, C_2, follows the parabolic path $y = x^2$ from $(1, 1)$ to $(2, 4)$.

   (a) (3 points) Parameterize the path C_1, including bounds on the parameter.

   (b) (2 points) Parameterize the path C_2, including bounds on the parameter.

   (c) (5 points) Compute

$$\int_C (x + 2y)\, dx + \sin y\, dy.$$

3. (15 points) Let E be the set

$$E = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + 4z^2 \leqslant 12\}.$$

Let $f(x, y, z) = xyz$. Find the absolute maximum and minimum values of $f(x, y, z)$ in E.

4. (10 points) Find parametric equations for the line of intersection of the planes P_1 and P_2:

$$P_1 : x + y + z = 2 \qquad P_2 : y - 3z + 6 = 0$$

5. (10 points) Consider the function

$$f(x, y, z) = \ln(1 + xyz) + e^z \cos(xy)$$

where

$$x = u \cos v, \qquad y = u \sin v \qquad z = u^2.$$

Find the value of $\frac{\partial f}{\partial u}$ when $u = 2$ and $v = 0$.

6. (5 points) Suppose that

$$\iint_D f(x, y)\, dA = 16$$

where D is the triangular region on xy-plane with vertices $(1, 0)$, $(3, 0)$ and $(2, 2)$. Find the average value of the function f over the region D.

7. (15 points) Let S be the boundary of the region of space E in the first quadrant, bounded above by $z = 4 - \sqrt{x^2 + y^2}$. Let S be oriented corresponding to outward facing normal vectors. Compute

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = (\sin z + e^x)\mathbf{i} + (1 - ye^x)\mathbf{j} + (3z - \cos(x^2))\mathbf{k}$.

8. (a) (2 points) Let $F = \langle P, Q \rangle$ and let C be a piecewise smooth curve bounding a closed region D of the plane. Explain what it means for C to be *positively oriented*.

   (b) (3 points) State Green's theorem for the situation described in part (a).

   (c) (10 points) Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the positively oriented boundary of the region D bounded above by $y = 4 - x^2$ and below by the x-axis, where

$$\mathbf{F}(x, y) = \left\langle -\frac{1}{2}y^2 + \cos y, \sqrt[3]{y^2 - 2} + x \sin y \right\rangle$$

9. (a) (15 points) Using curl, show that

$$\mathbf{F}(x,y,z) = (z^2 + ye^x)\mathbf{i} + (e^x - y\cos z)\mathbf{j} + (2zx + \frac{1}{2}y^2 \sin z)\mathbf{k}$$

is conservative.

(b) Find a potential function for $\mathbf{F}$.

(c) Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is given by $\langle 4\cos^3 t, 4\sin^3 t, t\rangle$ where t varies from 0 to π.

10. (10 points) Let $\mathbf{F} = \mathbf{F}(x, y, z)$ denote a vector field, and $f = f(x, y, z)$ denote a scalar function. Determine whether each of the expressions yields a vector field, a scalar function or has no meaning. Each part is worth 1 point.

    (a) $\operatorname{curl}(\nabla f)$

    (b) $\operatorname{div}(\operatorname{div} \mathbf{F})$

    (c) $\nabla(\nabla \times \mathbf{F})$

    (d) $\mathbf{F} \cdot (\nabla \times \mathbf{F})$

    (e) $\operatorname{curl}(\operatorname{curl} f)$

    (f) $\operatorname{div}(\operatorname{curl} \operatorname{grad} f)$

    (g) $\nabla(\mathbf{F} \cdot \nabla f)$

    (h) $\operatorname{curl}(f) \times \operatorname{div} \mathbf{F}$

    (i) $\nabla(\operatorname{div} \mathbf{F})$

    (j) $(\nabla \times \mathbf{F}) \cdot \nabla f$

11. (15 points) Using Stokes' Theorem, evaluate the outward flux of curl $\mathbf{F}$ i.e., $\iint_S \text{curl}\,\mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} =< x, y, y >$ and S is the non-closed surface consisting of the part of the cylinder $x^2 + y^2 = 4$ in $0 \leq z \leq 4$ and the part of plane $z = 4$ inside of the cylinder $x^2 + y^2 = 4$.

12. (15 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F} = (e^{-x} + \tan x)\mathbf{i} + x^2\mathbf{j} + (e^{z^2} + \cosh z)\mathbf{k}$$

and C boundary of the part of the plane $2x + 2y + z = 10$ in the first octant, oriented counterclockwise when viewed from above.

13. (5 points) Convert the triple integral $\iiint_E (x+z)\, dV$, where E is the solid region in xyz-space that lies above the cone $z = \sqrt{x^2+y^2}$ and inside of the sphere $x^2+y^2+z^2 = 9$, into spherical coordinates. DO NOT EVALUATE THIS INTEGRAL.