

## Review for Final Exam

MATH 230, FALL 2006

THE FINAL EXAM IS SCHEDULED ON 12/18/2006 12:20 PM - 2:10PM

This is a comprehensive exam, you should consult your previous midterm review sheets for highlights of Sections 13.1-16.6

### Important skills for sections 16.7 - 17.9

#### Section 16.7 - 16.8: Triple Integrals

- Evaluate triple integrals in rectangular coordinates by converting them to iterated integrals (Fubini's theorem);
- Recognize the solid from an iterated integral (in rectangular, cylindrical or spherical coordinates);
- Find the volume of a solid using the triple integral  $\iiint_E dV$ ;
- Change from rectangular coordinates to cylindrical coordinates in a triple integral;
- Change from rectangular coordinates to spherical coordinates in a triple integral;
- Change the order of integration.

#### Section 16.9: Change of Variables in Multiple Integrals

- Find the image of a planar region under the given transformation;
- Find the preimage under a given transformation;
- Find the Jacobian of a given transformation;
- Choose an appropriate transformation and evaluate the (double) integral.

#### Section 17.1: Vector Fields

- Definitions of the notions 'vector field', 'conservative vector field', 'potential function';
- Find the gradient vector field of a scalar function  $f(x, y)$  (or  $f(x, y, z)$ );
- Applications: velocity field, force field. Pictures of vector fields.

#### Section 17.2: Line integrals

- Find parameterizations of line segments and other simple piecewise smooth curves (be sure to specify the range of values of the parameter);
- Evaluate a line integral of a scalar function  $f(x, y, z)$  along a curve  $C$  given by  $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ ,  $a \leq t \leq b$  with respect to *arc length*;
- Evaluate a similar line integral with respect to  $x, y, z$ ;

- Evaluate a line integral with respect to arc length of the tangential component of a vector field along a curve (parameterized, smooth, as usual);
- All the same for the two-dimensional case;
- Find the work done by a force field in moving a particle along some curve (i.e., the line integral with respect to arc length of the tangential component of the force).

### Section 17.3: Fundamental Theorem for Line integrals

- Determine whether or not a vector field is *conservative* by using the test, if it can be applied. Note that the test is different for  $\mathbf{F}(x, y)$  and  $\mathbf{F}(x, y, z)$  and that it includes the notion of a simply connected region in the former case;
- Find a *potential function* for a conservative vector field;
- Evaluate a line integral (with respect to arc length) of (the tangential component of) a conservative vector field using the *fundamental theorem for line integrals*;
- Show that a line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  is *independent of path* by showing that the vector field  $\mathbf{F}$  is conservative. Same for the integral being zero along any closed path  $C$ .

### Section 17.4: Green's Theorem

- Determine the orientation of a simple closed planar curve;
- Use Green's Theorem to evaluate the line integral along a given curve (with positive/negative orientation);
- Use Green's Theorem to evaluate a double integral;
- Use Green's Theorem to compute the area of the planar region enclosed by a simple closed curve.

### Section 17.5: Curl and Divergence

- Find the *curl* of a vector field;
- Find the *divergence* of a vector field;
- Determine the sign of the divergence and the direction of the curl by a sketch of the vector field;
- Determine whether an expression is meaningful (refer to ex.17.5.12);
- Determine whether or not a vector field defined on all of  $\mathbb{R}^3$  is *conservative*;
- Find a potential function for a conservative vector field;
- Determine whether or not a vector field is *irrotational*;

- Determine whether or not a vector field is *incompressible*;
- Vector forms of the Green's Theorem. (Just look through. If don't understand it, probably you don't understand the regular version of the Green's Theorem either).

### Section 17.6: Parametric Surfaces and their Areas

- Find parametric equations (or a vector equation) for a surface (i.e. parameterize a surface). Make sure to specify the domain  $D$ ;
- Identify a surface by its parametric equations;
- Find the tangent plane at a given point to a parametric surface (a surface that is or can be parameterized). Note that the point can be given as  $(x, y, z)$ , then you may need to solve for the values of the parameters;
- Find the surface area of a smooth parametric surface  $S$ ;
- Find the surface area of the graph of a function ( $z = f(x, y)$ ,  $y = f(x, z)$  or  $x = f(y, z)$ );
- The natural way to parameterize the surfaces of revolution (along the axis of revolution).

### Section 17.7: Surface Integrals

- Evaluate the *surface integral* of a scalar function over a parametric surface  $S$  (or of a surface which is the graph of some function);
- Determine which orientation of the surface should be used and/or is used in some particular case. Distinguish between the orientation inwards/outwards, upwards/downwards — i.e. understand, how to interpret the information that a surface is, for example, oriented downwards);
- Find the flux of a given vector field across a given oriented surface.

### Section 17.8: Stokes' Theorem

- Use Stokes' Theorem to evaluate  $\iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S}$ ;
- Use Stokes' Theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ ;
- Pay attention to the orientations of the curve and the surface while applying the Stokes' Theorem. (Understand, what happens, if inconsistent orientations are chosen).

### Section 17.9: Divergence Theorem

- Use the Divergence Theorem to calculate the surface integral  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ ;
- Gauss' Law of electrostatics (mentioned in 17.7);
- Interpret the Divergence Theorem in terms of pictures of vector fields (see ex.17.9.1);
- Find in section 17.5 the two-dimensional counterpart of the Divergence Theorem.