Name: ______________________

MATH 231 Student Id Number: ______________________

Final Exam Instructor: ______________________

December 21, 2006 Section: ______________________

Instructions:

- This exam has 16 problems for a total of 150 points. There are **8** multiple choice problems and **8** partial credit problems. Each multiple choice problem is worth 5 points. The point value for each partial credit problem is in parentheses to the right of the problem number.
- To receive full credit, you must solve each problem on this exam fully and correctly.
- **In order to obtain full credit for partial credit problems, all work must be shown. No credit will be given for an answer or a step without supporting work.**
- Some questions have more than one part. Check carefully to ensure you don't miss any parts.
- The point value for each question is in parentheses to the right of the question number.
- Please check if all pages are in the exam set before you begin.
- **THE USE OF CALCULATORS, BOOKS, NOTES ETC IS NOT PERMITTED IN THIS EXAMINATION.**
- At the end of the examination, the booklet will be collected.

1. (5 points) Consider the planes

$$\text{P1:} \quad 3x + ay + 9z = 3 \qquad\qquad \text{P2:} \quad x - 2y + 3z = 1$$

where a is a constant. For what value of a will the two planes be parallel to each other?

(a) $a = 1$

(b) $a = -6$

(c) $a = 3$

(d) $a = 6$

(e) $a = -4$

2. (5 points) Consider the vectors $\overrightarrow{a} =< 3, 3, 0 >$, $\overrightarrow{b} =< 1, 0, 1 >$, $\overrightarrow{c} =< -1, 1, 0 >$. Decide which one of the following statements is true:

(a) $\overrightarrow{a}$ is perpendicular to $\overrightarrow{c}$

(b) $\overrightarrow{a} \cdot \overrightarrow{b} = 0$

(c) $\overrightarrow{b}$ is parallel to $\overrightarrow{c}$

(d) $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{0}$

(e) None of the above.

3. (5 points) Given the curve $\vec{r}(t) =< 2\cos(t), 2\sin(t), 0 >$. What is its curvature κ at $t = 0$?

   (a) $\kappa(0) = 2$

   (b) $\kappa(0) = 1$

   (c) $\kappa(0) = 0.5$

   (d) $\kappa(0) = -0.5$

   (e) None of the above.

4. (5 points) If $\mathbf{r}(t) = \langle \cos t, \cos t \rangle$, find the arc length from $t = 0$ to $t = \pi/2$.

   (a) 1

   (b) 2

   (c) $\pi/2$

   (d) $\sqrt{2}$

   (e) -1

5. (5 points) What is the domain of the function

$$f(x, y) = \frac{1}{\sqrt{x^2 + y^2 - 9}} \text{ ?}$$

(a) $\{(x, y) \mid x^2 + y^2 > 9\}$

(b) $\{(x, y) \mid x^2 + y^2 > 3\}$

(c) $\{(x, y) \mid x^2 + y^2 < 9\}$

(d) $\{(x, y) \mid x^2 + y^2 \geq 0\}$

(e) $\{(x, y) \mid x^2 + y^2 > 0\}$

6. (5 points) What is the range of the function

$$f(x, y) = \frac{1}{\sqrt{x^2 + y^2 + 9}} \text{ ?}$$

(a) $f(x, y) \geq 0$

(b) $0 \leq f(x, y) \leq 3$

(c) $0 < f(x, y) \leq \frac{1}{3}$

(d) $f(x, y) \geq \frac{1}{3}$

(e) None of the above.

7. (5 points) Given the equation $z^2 + y^3 = xy^2 + 2$. Find $\dfrac{\partial z}{\partial x}$.

(a) $\dfrac{\partial z}{\partial x} = \dfrac{y^2}{2z - x^2}$

(b) $\dfrac{\partial z}{\partial x} = \dfrac{y^2}{2z}$

(c) $\dfrac{\partial z}{\partial x} = \dfrac{y^2 - 2xz}{2z}$

(d) $\dfrac{\partial z}{\partial x} = \dfrac{2xz}{2z + x^2}$

(e) $\dfrac{\partial z}{\partial x} = \dfrac{2z}{y^2 - 2xz}$

8. (5 points) The gradient vector ∇f of $f(x, y) = \sin(xy) + e^{2x} + y$ is:

(a) $< y\cos(xy) + 2e^{2x}, x\cos(xy) + 1 >$

(b) $< \cos(xy) + e^{2x}, \cos(xy) + 1 >$

(c) $< \cos(xy) + 2e^{2x}, \cos(xy) + 1 >$

(d) $< y\cos(xy) + e^{2x}, x\cos(xy) + 1 >$

(e) $< y\cos(xy) - 2e^{2x}, x\cos(xy) - 1 >$

9. (14 points) Do the following:

   (a) (6 points) Find parametric equations of the line L through the points $(1, 0, 1)$ and $(4, -2, 2)$.

   (b) (6 points) Find the point of intersection of the line L in part (a) and the plane $2x+y+z = 5$.

10. (12 points) A particle has acceleration $\mathbf{a}(t) = \langle\ 2\ ,\ \dfrac{-1}{(t+1)^2}\ \rangle$. If the velocity at $t = 1$ is $\langle\ 0\ ,\ \dfrac{3}{2}\ \rangle$, and the position at $t = 0$ is $\langle 3, 0 \rangle$, find the position of the particle at $t = 1$.

(Hint: You <u>might</u> need: $\dfrac{d}{dt}\left(\dfrac{-1}{(t+1)^2}\right) = \dfrac{2}{(t+1)^3}$ or $\displaystyle\int \dfrac{-1}{(t+1)^2} = \dfrac{1}{t+1} + C$.)

11. (12 points) Do the following:

(a) (6 points) Find the limit, if it exists, or show that the limit doesn't exist.

$$\lim_{(x,y)\to(0,0)} \frac{2x^2 + \sin^2(y)}{x^2 + y^2}$$

(b) (6 points) Determine the set of points at which the function $f(x, y)$ is continuous:

$$f(x, y) = \frac{9}{\sqrt{x + y} - 3}.$$

12. (12 points) Show $u(x, y) = e^y \sin x$ is a solution of Laplace's equation $u_{xx} + u_{yy} = 0$. Following the steps:

(a) (5 points) Find u_x and u_y.

(b) (5 points) Find u_{xx} and u_{yy}.

(c) (2 points) Show that $u(x, y)$ is a solution of Laplace's equation $u_{xx} + u_{yy} = 0$.

13. (12 points) Let $f(x, y) = \sqrt{x + e^{2y}}$.

(a) (9 points) Find the linearization $L(x, y)$ at the point $(3, 0)$.

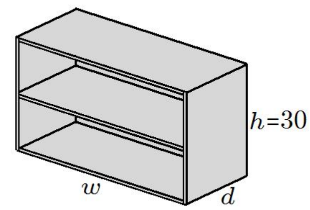
(b) (3 points) Use it to approximate $f(2.96, 0.06)$.

14. (18 points) Identify **all** local maximum points, local minimum points and saddle points of the function $f(x, y) = x^3 + 12xy^2 - 12x$.

15. (12 points) If $z = x^2 + y^2 + x + y + 2$, where $x = r\cos\theta$ and $y = r\sin\theta$, find $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial \theta}$ at $\theta = 0$ and $r = 2$.

16. (18 points) A small bookcase with one middle shelf (see figure below) has volume $V = dwh$ and the area of the wood board is $A = wh + 2dh + 3wd$. The bookcase needs to have volume $6,000$ cubic inches, and the height of the bookcase is $h = 30$ inches. Use Lagrange multipliers to find the width w and depth d that will minimize the amount of wood required.

    You must use Lagrange multipliers to receive credit for your work.



Please do not write here. For instructor use only.

Problem number	Full score	Your score
(1)	5	
(2)	5	
(3)	5	
(4)	5	
(5)	5	
(6)	5	
(7)	5	
(8)	5	
(9)	14	
(10)	12	
(11)	12	
(12)	12	
(13)	12	
(14)	18	
(15)	12	
(16)	18	
Total	150	