Name: ____________________

MATH 231 | Student Id Number: ____________________

Midterm Exam | Instructor: ____________________

November 1, 2006 | Section: ____________________

Instructions:

- This exam has 10 partial credit questions for a total of 100 points. Each problem is worth 10 points.
- To receive full credit, you must solve each problem on this exam fully and correctly.
- **In order to obtain full credit for partial credit problems, all work must be shown.**
- **Credit will not be given for an answer not supported by work.**
- Some questions have more than one part. Check carefully to ensure you don't miss any parts.
- The point value for each question is in parentheses to the right of the question number.
- Please check if all pages are in the exam set before you begin.
- **THE USE OF CALCULATORS, BOOKS, NOTES ETC IS NOT PERMITTED IN THIS EXAMINATION.**
- At the end of the examination, the booklet will be collected.

1. Do the following problems:

   (a) (2 points) Find the distance between the points $P(1, 2, 3)$ and $Q(3, 5, 7)$.

   (b) (3 points) Write the equation of the sphere of radius 3 centered at $P(1, 2, 3)$.

   (c) (5 points) Show that $x^2 + y^2 + z^2 + 2x - 4y + 8z + 6 = 0$ is the equation of a sphere and find its center and radius.

2. Let $\overrightarrow{a} =< 2, 1, 3 >$ and $\overrightarrow{b} =< -1, 2, 1 >$. Find the following:

   (a) (3 points) $|\overrightarrow{a}|$

   (b) (4 points) $\overrightarrow{a} - 3\overrightarrow{b}$

   (c) (3 points) A unit vector in the direction of $\overrightarrow{a}$

3. Find the equation of the plane through the points $P(1,2,0)$, $Q(0,1,0)$ and $R(1,1,1)$. Follow the steps:

   (a) (2 points) $\overrightarrow{PQ} =$

   $\overrightarrow{PR} =$

   (b) (4 points) $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} =$

   (c) (4 points) Write the equation of the plane through $P = (1,2,0)$ with normal $\vec{n}$.

4. Match the following equations with the graphs of the surfaces they correspond to. (You don't need to explain.)
   (a) (2 points) $z = 2y^2 - 2x^2$ matches plot ________________
   (b) (2 points) $x^2 + y^2 = 1$ matches plot ________________
   (c) (2 points) $z^2 = x^2 + y^2 + 1$ matches plot ________________
   (d) (2 points) $z = 2x^2$ matches plot ________________
   (e) (2 points) $4x^2 + y^2 + z^2 = 1$ matches plot ________________

5. Let $\mathbf{a} =< 2, 2, -1 >$ and $\mathbf{b} =< 4, 1, 1 >$ be two given vectors, and let θ be the angle between them. Find the following:

(a) (3 points) $\mathbf{a} \cdot \mathbf{b}$

(b) (3 points) $\cos\theta$

(c) (4 points) Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$, i.e., $\text{comp}_{\mathbf{a}}\mathbf{b}$

6. Check if the following four points lie in the same plane:

$$P(1, 1, -2), \quad Q(0, -3, 3), \quad R(2, -1, -1) \quad \text{and} \quad S(3, 2, -5).$$

Follow the steps:

(a) (3 points) $\vec{a} = \overrightarrow{PQ} =$

$\vec{b} = \overrightarrow{PR} =$

$\vec{c} = \overrightarrow{PS} =$

(b) (4 points) $\vec{b} \times \vec{c} =$

(c) (2 points) $\left|\vec{a} \cdot (\vec{b} \times \vec{c})\right| =$

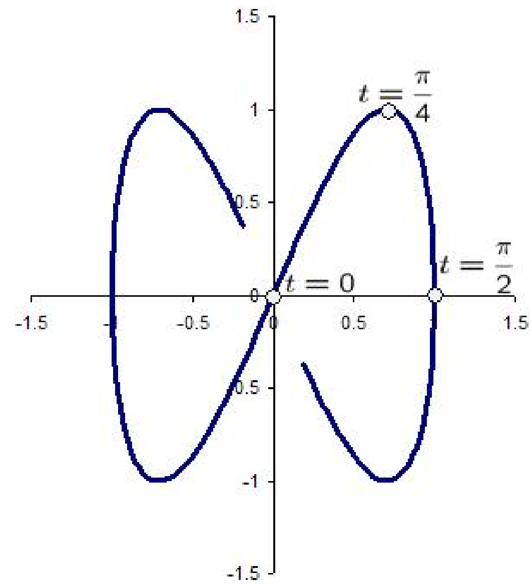
(d) (1 point) Do these four points lie in a plane? Why?

7. Consider the curve represented by the vector function

$$\mathbf{r}(t) = \langle \ \sin t \ , \ \sin 2t \ , \ 0 \ \rangle \, .$$

(See the given graph.) We consider three points on the curve, at

$$t = 0, \quad t = \frac{\pi}{4} \quad \text{and} \quad t = \frac{\pi}{2}.$$



(a) (3 points) Without actually computing the curvatures, judging only by the graph, decide which of the three points has the greatest curvature. Give reason.

(b) (7 points) Compute the curvature at that point.

8. Consider a vector function $\mathbf{r}(t) = \langle\ t^2 + 4t + 6\ ,\ 3t - 6\ ,\ t^2 - 4t + 3\ \rangle$.

(a) (5 points) For what value(s) of t is the tangent vector perpendicular to $\mathbf{k}$?

(b) (5 points) Find the unit tangent vector at the point $(3, -9, 8)$.

9. (10 points) Find the arc length of the curve

$$x = \sin t - t\cos t, \qquad y = \cos t + t\sin t, \qquad z = t^2$$

from $t = 0$ to $t = 4$.

10. (10 points) Let $\mathbf{a}(t) = \left\langle\ 2\ ,\ \pi^2 \sin(\pi t)\ \right\rangle$ be the acceleration function for a moving particle in a plane. If the velocity at $t = 0$ is $\mathbf{v}(0) = \langle 4, -\pi \rangle$, and the position at $t = 1$ is $\mathbf{r}(1) = \langle -2, 2 \rangle$, find the position at $t = 0$, i.e., $\mathbf{r}(0)$.

Please do not write here. For instructor use only.

Problem number	1	2	3	4	5	6	7	8	9	10	Total
Points	10	10	10	10	10	10	10	10	10	10	100
Your points											

Answer keys to midterm exam MATH231 Fall 2006

Wen Shen

November, 2006

1. (a) $d = \sqrt{29}$

   (b) $(x-1)^2 + (y-2)^2 + (z-3)^2 = 9$

   (c) $(x+1)^2 + (y-2)^2 + (z+4)^2 = 15$, so center is $(-1, 2, -4)$, radius is $\sqrt{15}$.

2. (a) $\sqrt{14}$

   (b) $< 5, -5, 0 >$

   (c) $< \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}} >$

3. (a) $< -1, -1, 0 >$, and $< 0, -1, 1 >$

   (b) $< -1, 1, 1 >$

   (c) $-x + y + z = 1$

4. (a) 9, (b) 1, (c) 4, (d) 2, (e) 8

5. (a) 9,

   (b) $\sqrt{2}/2$,

   (c) 3

6. (a) $< -1, -4, 5 >$, $< 1, -2, 1 >$, $< 2, 1, -3 >$

   (b) $< 5, 5, 5 >$

   (c) 0

   (d) yes, they are coplanar because teh triple product is zero.

7. (a) At $t = \frac{\pi}{4}$ because the curve bends most there.

   (b) $\kappa(\frac{\pi}{4}) = 8$.

8. (a) $r'(t) =< 2t+4, 3, 2t-4 >$, and $r'(t) \cdot \mathbf{k} = 0$ gives $2t - 4 = 0$, i.e., $t = 2$.

   (b) $T(t) = \frac{r'(t)}{|r'(t)|}$, and $t = -1$ at the point $(3, -9, 8)$, which leads to $T(-1) =< \frac{2}{7}, \frac{3}{7}, -\frac{6}{7} >$.

9. $r'(t) =< t\sin t, t\cos t, 2t >$, $|r'(t)| = \sqrt{5}t$, then $L = \int_0^4 |r'(t)|\, dt = \int_0^4 \sqrt{5}t\, dt = 8\sqrt{5}$.

10. $v(t) =< 2t+4, -\pi\cos\pi t >$, and $r(t) =< t^2 + 4t - 7, -\sin\pi t + 2 >$, so $r(0) =< -7, 2 >$.