

JULY 13, 05

1. Consider the vectors $\vec{u} = \hat{i} - 2\hat{j}$ and $\vec{v} = -6\hat{i} + 5\hat{j} - 2\hat{k}$.
 - a) Find the unit vectors in the directions of \vec{u} and \vec{v} .
 - b) Find a vector of length 10 in the same direction as \vec{u} and another of length 2 in the **opposite** direction of \vec{v} .
2. Which of the following vectors are parallel? Which are perpendicular? Find the cosine of the angle between \vec{u} and \vec{v} .
 $\vec{u} = \langle -2, 3, 2 \rangle$
 $\vec{v} = \langle 0, -4, 1 \rangle$
 $\vec{w} = \langle 7, 2, 8 \rangle$
 $\vec{x} = \langle 2, -3, -1 \rangle$
 $\vec{y} = \langle 6, -9, -6 \rangle$
3. Let $\vec{a} = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} + \hat{k}$.
 - a) Find the vector projection of \vec{a} onto \vec{b} .
 - b) Find the vector component of \vec{a} that is orthogonal to \vec{b} .
 - c) Find a vector that is orthogonal to both \vec{a} and \vec{b} .
4. a) Find the area of the parallelogram with vertices $P = (2, -1, 1)$, $Q = (5, 1, 4)$, $R = (0, 1, 1)$, and $S = (3, 3, 4)$ (Hint: $\vec{PQ} = \vec{RS}$ and $\vec{PR} = \vec{QS}$).
b) Find the volume of the parallelepiped formed by the vectors $\vec{u} = \langle -3, -1, -1 \rangle$, $\vec{v} = \langle 3, 2, 3 \rangle$, and $\vec{w} = \langle -2, 2, 0 \rangle$.

5. Consider the line L through the points $(1, -2, -4)$ and $(-3, 0, 2)$.

- a) Is the point $(-7, 2, 8)$ on L ?
- b) Find an equation for the line through the point $(1, 1, 1)$ that is parallel to L .

6. FIND

EQUATION:

- a) Plane through the points $(-1, 0, 3)$, $(2, -2, 3)$, and $(3, 4, 0)$.
- b) Plane through the point $(-1, 3, 1)$ and perpendicular to the line $x = -2t$, $y = -3 + 5t$, $z = 2 - 7t$. Also, find the point of intersection of the line with this plane.

7. Consider the planes $x - 3y + 6z = 4$ and $5x + y - z = 4$.

- a) What are the normal vectors for each plane?
- b) Find the line of intersection of the planes.
- c) Find the distance from the first plane to the point $(0, 0, 0)$.

8. Convert the points and equations from the listed coordinate system to the other.

- a) $(\rho, \theta, \phi) = (9, \pi, \frac{\pi}{4})$ from spherical to rectangular.

- b) $(r, \theta, z) = (3, \frac{3\pi}{2}, -2)$ from cylindrical to rectangular.

- c) $x^2 + y^2 = 4y$ from rectangular to cylindrical.

- d) $\rho = 4 \sec \phi$ from spherical to rectangular.

9. Extra -

- a) If two planes are parallel, what is the relationship between their normal vectors? What if the planes are perpendicular?

- b) Find an equation for the plane that passes through $(2, 2, 1)$ and $(-1, 1, -1)$ and is perpendicular to the plane $2x - 3y + z = 3$ (Hint: find 2 vectors to which the normal vector of the plane in question is orthogonal).