

MATH 251  
Midterm Exam I  
Fall 2002

NAME \_\_\_\_\_  
STUDENT NUMBER \_\_\_\_\_  
INSTRUCTOR \_\_\_\_\_

There are ?? questions on ?? pages. Please read each problem carefully before starting to solve it. Show all work – **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.

**Point - grader use only**

1. ( 8 pt) \_\_\_\_\_
2. (10 pt) \_\_\_\_\_
3. (12 pt) \_\_\_\_\_
4. (12 pt) \_\_\_\_\_
5. (10 pt) \_\_\_\_\_
6. (12 pt) \_\_\_\_\_
7. (12 pt) \_\_\_\_\_
8. (12 pt) \_\_\_\_\_
9. (12 pt) \_\_\_\_\_
- Total \_\_\_\_\_

1. (8 points) (a) Classify the following differential equations as linear or non-linear and state their order.

	linear/ non-linear	order
$t^2 \frac{d^2 y}{dt^2} + 3t \frac{dy}{dt} - y \sin t = 0$		
$2y' - \sin y = 0$		
$y''' - 3y' + y + t = 0$		

- (b) Which of the **linear** differential equations above are homogeneous?

2. (10 points) Find the general solution.

(a)  $y'' - 4y' + 13y = 0$

(b)  $y'' + 2y' + y = 0$

3. (12 points) Find the **explicit** solution of the initial value problems

(a)  $y' = \frac{\cos t + 1}{y}$ ,  $y(0) = -3$ .

(b)  $ty' + 3y = 3$ ,  $y(1) = 2$ .

4. (12 points) A college student owes \$1000 to a credit card company, which charges interest at an annual rate of 10%. The student makes payments continuously at a constant rate of \$10/month (\$120/year).

(a) Set up the initial value problem describing the situation.

(b) Solve the initial value problem of part (a).

(c) Find the time  $T$  it will take to pay off the debt.

5. (10 points) (a) What is the largest interval on which a solution of the initial value problem is guaranteed to exist? Do **not** solve the differential equation.

$$y'' + \ln|t - 10|y' + y = \frac{1}{t^2 - t}, \quad y(8) = 8$$

- (b) For which values  $(t_0, y_0)$  of the initial condition is the solution of the initial value problem guaranteed to be unique?. Do **not** solve the differential equation.

$$y' = (1 - t^2 - y^2)^{1/2}, \quad y(t_0) = y_0$$

6. (12 points) Consider the differential equation:

$$y' = y(9 - y^2)$$

(a) Find the critical (equilibrium) points.

(b) Classify the critical points as stable, unstable or semistable. Provide a justification.

(c) If  $y(0) = -2$ , what is the limit of  $y$  as  $t$  goes to  $\infty$ ?

7. (12 points) Consider the differential equation:

$$(e^x \sin y + x^2) + (e^x \cos y + y)y' = 0$$

(a) Show that the above equation is exact.

(b) Give the general solution to the equation.

8. (12 points) (a) Find the general solution of

$$y'' + y' - 2y = 0$$

(b) Given the initial value problem

$$y'' + y' - 2y = 0, \quad y(0) = \beta, \quad y'(0) = 1$$

What value(s) of  $\beta$  will make  $\lim_{t \rightarrow \infty} y(t) = 0$ .

9. (12 points) (a) Show that the functions

$$y_1(t) = t^2, \quad y_2(t) = t^{-2}$$

are solutions of the differential equation

$$t^2 y'' + t y' - 4y = 0$$

(b) Are they a fundamental set of solutions? Justify.