

There are 5 multiple choice and 7 partial credit problems in this examination. Each multiple choice problem has 5 choices. Circle the correct answer. Each multiple choice problem is worth 5 points.

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK YOUR EXAMINATION BOOKLET CAREFULLY.
THERE SHOULD BE 12 PROBLEMS ON 10 PAGES.

1. (5 points) Which of the following is TRUE?

- a) $y' = \frac{t}{y}$ is a first order linear differential equation
- b) $\sin t y'' + (1 - t^2)y' + \cos t y = 0$ is a second order linear differential equation
- c) $y'' + (y')^3 + y = 0$ is a nonlinear differential equation of order 3
- d) $y'' + y' + y = t$ is a second order homogeneous differential equation
- e) $\frac{\partial y}{\partial t} + ty = 0$ is an ordinary differential equation

2. (5 points) Let $y(t)$ be the solution of the initial value problem

$$\begin{aligned}\frac{dy}{dx} &= y^3 - y \\ y(0) &= \frac{9}{10}.\end{aligned}$$

Then $\lim_{t \rightarrow \infty} y(t)$ is equal to

- a) ∞
- b) $\frac{9}{10}$
- c) 1
- d) 0
- e) -1

3. (5 points) Find the value for the constant b , for which given equation is exact.

$$(e^x \sin y + bx^2y^2)dx + (e^x \cos y + x^3y)dy = 0$$

a) $b = 0$

b) $b = \frac{1}{3}$

c) $b = 3$

d) $b = \frac{3}{2}$

e) $b = 1$

4. (5 points) Let $y_1(t)$ and $y_2(t)$ be two solutions of a second order, homogeneous, linear differential equation. Suppose the Wronskian $W(y_1(t), y_2(t)) = e^{-t}$. Which of the following is FALSE?

a) $y_1(t)$ and $y_2(t)$ are linearly independent functions.

b) $2y_1(t) - 3y_2(t)$ is also a solution of the differential equation.

c) $y_1(t)$ and $y_2(t)$ do not constitute a fundamental set of solutions.

d) All solutions of the differential equation can be expressed as $c_1y_1(t) + c_2y_2(t)$, where c_1 and c_2 are constants.

e) $W(2y_1(t), 3y_2(t)) = 6e^{-t}$

5. (5 points) The largest interval on which the differential equation

$$(t^2 - 1)y'' + \sin t y' + \cos t y = 0, \quad y(5) = 0, \quad y'(5) = 1$$

is certain to have a unique twice differentiable solution is

- a) $(-\infty, 5)$
- b) $(-1, 1)$
- c) $(5, \infty)$
- d) $(1, \infty)$
- e) $(-\infty, -1)$

6. (5 points) Find the solution to the following initial value problem in **explicit form**.

$$\begin{aligned} \frac{dy}{dx} &= -\frac{x}{y} \\ y(0) &= -1 \end{aligned}$$

7. (12 points) Find the general solution of

$$t^2 \frac{dy}{dt} + 3ty = e^t, \quad t > 0$$

8. (13 points) Find the general solution of

$$(2xy - 3x^2)dx + (x^2 + 2y)dy = 0$$

(you may keep your solution in implicit form).

9. (12 points) Find the general solution of the following equations. Express your answer in terms of real valued functions.

a) $y'' - 4y' + 4y = 0$

b) $y'' - 4y' + 5y = 0$

10. (8 points) Solve the initial value problem

$$\begin{cases} y'' - 4y = 0 \\ y(0) = 4, \quad y'(0) = 4 \end{cases}$$

11. (12 points) A jar contains a sugar solution. Initially it had 2 liters of water and 15 grams of dissolved sugar. Water containing 5 grams of sugar per liter enters the jar at a rate of 2 liters/min. The well stirred mixture flows out at the same rate.
How many grams of dissolved sugar is present in the jar after $\ln 5$ minutes?

12. (13 points) A ball of mass 2 kg is dropped from rest in a viscous liquid. As a result the ball experiences a drag force which is twice the magnitude of its velocity. Find the distance the ball travels in the first 2 seconds of its motion. Assume that $g = 10 \text{ m/sec}^2$.