

MATH 251
Fall 2003
Exam 1
October 14, 2003

ANSWERS:

1. First order, linear;
Second order, linear;
First order, nonlinear;
Third order, nonlinear.

2. C

3. A

4. D

5. $y(t) = -4 + \sqrt{x^4 - 3x^2 + 3x + 3}$.

6. (a) $\frac{\partial}{\partial y}(2x + ye^{xy}) = e^{xy} + xye^{xy} = \frac{\partial}{\partial x}(xe^{xy} + 1)$

(b) $x^2 + e^{xy} + y = 2$

7. (a) Equilibrium solutions are $y = -3$, $y = 0$, and $y = 2$.

(b) $y = -3$ is unstable, $y = 0$ is (asymptotically) stable, $y = 2$ is unstable.

(c) $\lim_{t \rightarrow \infty} y(t) = 0$

(d) $\lim_{t \rightarrow \infty} y(t) = -3$

8. (a) $Q(t) = 25000 - 20000e^{-\frac{1}{100}t}$

(b) The limiting concentration is $\lim_{t \rightarrow \infty} \frac{Q(t)}{500} = \frac{25000}{500} = 50 \left(\frac{g}{m^3}\right)$, which is the same as the concentration of the inflow.

9. $y(t) = 2e^{3t} - te^{3t}$

10. $y(t) = C_1t + C_2t^4$