MATH 251
Summer 2006
Exam I
June 29, 2006

ANSWERS:

1. (a) Third order, linear; (b) First order, nonlinear; (c) Second order, nonlinear.

2. C

3. C

4. B

5. A

6. (a) $y(t) = t^2 \sin t + \dfrac{2}{\pi^2} t^2$; (b) $(0, \infty)$.

7. (a) $y = 6, 5, -5$; (b) $y = -5$ is (asymptotically) stable, $y = 5$ is unstable, $y = 6$ is semistable;

(c) $\lim\limits_{t \to \infty} y(t) = -5$; (d) $y_0 = 5$.

8. (a) $\dfrac{\partial M}{\partial y} = -2x + e^{x+y} = \dfrac{\partial N}{\partial x}$; (b) $2x^3 - x^2 y + e^{x+y} = 4$.

9. $y(t) = 3e^{-2t} \cos 3t - e^{-2t} \sin 3t$

10. $y(t) = C_1 t + C_2 \, t \ln t$

11. (a) $Q' = 100 - \dfrac{1}{200} Q, \qquad Q(0) = 0$; (b) $Q(t) = 20000 - 20000 e^{\frac{-t}{200}}$;

(c) $t = 200 \ln 2$; (d) $\lim\limits_{t \to \infty} Q(t) = 20000 \; g$.