Final Exam Review

7:00-9:00pm, Dec 17; Willard 62

- You can bring **two** double-sided cheating sheets and a calculator with you in the exam.
- The exam is comprehensive, consisting of five written problems. But Chapter 3-5 will be the emphasis.
- Please go through all examples discussed in class and the homework problems.
- The cumulative Binomial, Poisson and Standard Normal probability tables will be provided.

Testing Points:

For both **Discrete** and **Continuous** r.v.:

- Joint and marginal probability mass function/density
- Cumulative Distribution Function (CDF):

$$F(x) = P(X \leq x) = \begin{cases} \sum_{y:y\leq x} p(y) \\ \int_{-\infty}^{x} f(y)dy \end{cases}$$

- Expected Value:

$$E(X) = \mu_x = \begin{cases} \sum_{x\in D} x \cdot p(x) \\ \int_{-\infty}^{\infty} x \cdot f(x)dx \end{cases}$$

- Variance: $V(X) = E[(X-\mu)^2] = E(X^2) - (EX)^2$
- Moment Generating Function (MGF):

$$M_X(t) = E(e^{tX}) = \begin{cases} \sum_{x\in D} e^{tx} \cdot p(x) \\ \int_{-\infty}^{\infty} e^{tx} \cdot f(x)dx \end{cases}$$

- Conditional PMF/density

$$p_{Y|X}(y|x) = \frac{p(x,y)}{p_X(x)}$$
$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$$

- Conditional mean and variance
- Covariance

$$\begin{aligned} Cov(X,Y) &= E[(X-\mu_X)(Y-\mu_Y)] \\ &= \begin{cases} \sum_x \sum_y (x-\mu_X)(y-\mu_Y)p(x,y) \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\mu_X)(y-\mu_Y)f(x,y)dxdy \end{cases} \end{aligned}$$

- Correlation $\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$
- Independence:

$$p(x,y) = p_X(x) \cdot p_Y(y) \text{ or } f(x,y) = f_X(x) \cdot f_Y(y)$$

- $E(h(X))$ or $E(g(X,Y))$

Important Distributions:

1. Binomial
2. Geometric
3. Normal
4. Uniform
5. Gamma
6. Exponential

Sample Questions:

Example 1: Suppose X is a discrete random variable taken values -1,0 and 1 with probability 2/5,1/5 and 2/5

a) Find the pmf(probability mass function) of X

b) Find $F(-2), F(0), F(1.5)$ where $F(x)$ is the cdf of X

c) Find mgf $M_X(t)$ of X

d) Find $E(X), E(3+2X), Var(X), Var(3+2X)$ and and $M_{(3+2X)}(t)$

e) Find $P(X=0|X>-1)$

Example 2: The joint frequency function of two discrete random variables, X and Y, is given in the following table:

x	y = 0	y = 1
0	$\frac{2}{7}$	$\frac{1}{7}$
1	$\frac{1}{7}$	$\frac{3}{7}$

a) Find the marginal pmf of Y
b) Find the conditional pmf of Y given X=1
c) Find $E(Y \mid X = 1)$
d) Find $Cov(X, Y)$, ρ_{XY} and $E(X + Y)$
e) Are X and Y independent?

Example 3: Suppose $X \sim N(1, 4)$, Find
a)$P(3 < X < 5)$
b)Find the value 'a' such that $P(X < a) = 0.95$

Example 4: Suppose X and Y have the joint density function

$$f(x, y) = 4xy, 0 \leq x \leq 1, 0 \leq y \leq 1$$

a) Find the marginal pdf of X and Y
b) Find $P(2X + Y > 1.5)$ and $P(Y < \frac{1}{2})$
c) Find the conditional density of Y given X
d) Are X and Y independent?