Midterm Exam 2 Review

1:25-2:15pm, Nov,10; Room 60 Willard

- You can bring a double-sided cheating sheet and a calculator with you in the exam.
- The exam covers all sections in Chapter3 and consists of five written problems.
- Please go through all examples discussed in class and the homework problems.
- Be familiar with the derivative computation in calculus.
- The cumulative Binomial and Poisson probability tables will be provided.

Concepts:

- Probability Distribution Function (PDF)/ Probability Mass Function (PMF): $P(X = x)$
- Cumulative Distribution Function (CDF): $F(x) = P(X \leq x)$
- Expected Value: $E(X) = \mu_x = \sum_{x \in D} x \cdot p(x)$
- Variance: $V(X) = \sum_D (x - \mu)^2 \cdot p(x) = E[(X - \mu)^2]$
- Moment Generating Function (MGF): $M_X(t) = E(e^{tX}) = \sum_{x \in D} e^{tx} p(x)$

Propositions and Theories

- $P(a \leq X \leq b) = F(b) - F(a - 1)$
  $P(X = a) = F(a) - F(a - 1)$
- $E[h(x)] = \sum_D h(x) \cdot p(x)$
  In particular, $E(aX + b) = a \cdot E(X) + b$
- A shortcut formula for variance: $V(X) = E(X^2) - [E(X)]^2$

- $V(aX+b) = \sigma^2_{aX+b} = a^2 \cdot \sigma^2_x$
- $E(X^r) = M^{(r)}_X(0)$
- Let $Y = aX + b$. Then $M_Y(t) = e^{bt}M_X(at)$.

<u>Distribution Summary:</u>

1. Bernoulli
   - Parameter: $\alpha, \quad 0 < \alpha < 1$
   - PMF:
   $$p(x;\alpha) = \begin{cases} 1-\alpha & \text{if } x = 0 \\ \alpha & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$
   - $E(X) = \alpha, Var(X) = \alpha(1-\alpha)$
   - $M_X(t) = 1 - \alpha + \alpha e^t$

2. Geometric
   - Parameter: $p, \quad 0 < p < 1$
   - PMF:
   $$p(x) = \begin{cases} (1-p)^{x-1}p & \text{if } x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$
   - $E(X) = 1/p, Var(X) = \frac{1-p}{p^2}$
   - $M_X(t) = \frac{pe^t}{1-(1-p)e^t}$

3. Binomial
   - Parameter: $n, p, \quad 0 < p < 1$
   - PMF:
   $$b(x;n,p) = \begin{cases} \binom{n}{x}p^x(1-p)^{n-x} & x = 0, 1, 2, \ldots n \\ 0 & \text{otherwise} \end{cases}$$
   - $E(X) = np, Var(X) = np(1-p)$
   - $M_X(t) = (1 - p + pe^t)^n$

4. Hypergeometric
   - Parameter: n, M, N
   - PMF:
$$h(x; n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$
   - $E(X) = n \cdot \frac{M}{N} \quad V(X) = (\frac{N-n}{N-1}) \cdot n \cdot \frac{M}{N} \cdot (1 - \frac{M}{N})$

5. Negative Binomial
   - Parameter: $r, p, \quad 0 < p < 1$
   - PMF:
$$nb(x; r, p) = \binom{x+r-1}{r-1} p^r (1-p)^x \quad x = 0, 1, 2, \ldots$$
   - $E(X) = \frac{r(1-p)}{p} \quad V(X) = \frac{r(1-p)}{p^2}$
   - $M_X(t) = \frac{p^r}{[1-e^t(1-p)]^r}$

6. Poisson
   - Parameter: $\lambda, \quad 0 < \lambda < 1$
   - PMF:
$$p(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2, \ldots$$
   - $E(X) = \lambda \quad V(X) = \lambda$
   - $M_X(t) = e^{\lambda(e^t - 1)}$

<u>Some useful derivative rules:</u>

- $(e^x)' = e^x \quad (lnx)' = \frac{1}{x} \quad (x^n)' = nx^{n-1} \quad (\frac{1}{x})' = -\frac{1}{x^2}$
- $(u \pm v)' = u' \pm v' \quad (u.v)' = u'.v + u.v' \quad (\frac{u}{v})' = \frac{u'.v - u.v'}{v^2}$
- Chain rule: $(f(u(x)))' = f'_u \cdot u'_x$

Practice exercises:

- Page 132: # 59
- Page 141: # 81 (a)(b)
- Page 147: # 95 (a) (b) (c)
- Page 151: # 131 (a) (b) (c) (d)