

# ECON 4323/5301 Homework 1

Fall 2007

Due September 20, 2007 at 5:30 pm

1. Let  $k = 7$ ,

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{pmatrix}, \text{ and } B = \begin{pmatrix} 0 & 1 & -1 \\ 4 & -1 & 2 \end{pmatrix}.$$

Calculate  $A + B$ ,  $B - A$ ,  $kB$ ,  $A^t$ ,  $AB$ , and  $AB^t$ , if possible.

2. Find the determinant and the inverse of each of the following matrices:

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{pmatrix}, \text{ and } C = \begin{pmatrix} 2 & 6 & 0 & 5 \\ 6 & 21 & 8 & 17 \\ 4 & 12 & -4 & 13 \\ 0 & -3 & -12 & 2 \end{pmatrix}.$$

3. Calculate the solution to the following system of linear equations:

$$\begin{aligned} 2x + 2y - z &= 2 \\ x + y + z &= -2 \\ 2x - 4y + 3z &= 0. \end{aligned}$$

4. Determine the definiteness of the following symmetric matrices:

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}, B = \begin{pmatrix} -3 & 4 \\ 4 & 4 \end{pmatrix}, C = \begin{pmatrix} -3 & 4 \\ 4 & -6 \end{pmatrix},$$

$$D = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 1 & -2 \\ 3 & -2 & 8 \end{pmatrix}, E = \begin{pmatrix} -3 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & -5 \end{pmatrix}, F = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{pmatrix}.$$

5. Find the first and second derivatives of the following functions:

(a)  $f(x) = x^7 + 3x^6 - 4x^2 + 5$

(b)  $f(x) = (3x^3 - 1)(x^2 + 7)$

(c)  $f(x) = (x^2 - 1)/(x^2 + 1)$

(d)  $f(x) = (x^3 - 4x^2 + 1)^5$

(e)  $f(x) = 3x^{2/3} + 3x^{-1}$

(f)  $f(x) = e^{x^2+3x}$

(g)  $f(x) = \ln(x^2 + 4)$

6. For the following functions, find the slope of the tangent line at  $x=5$  and determine the intervals where  $f$  is increasing:

(a)  $f(x) = 3x^2 - 12x + 13$

(b)  $f(x) = 2x/(x^2 + 2)$

7. Let  $f(x) = 10x^a$ , where  $a > 0$ . For what values of  $a$  is  $f$  convex? Concave?

8. Find all the first- and second- order partial derivatives of the following functions:

(a)  $f(x, y, z) = 3xyz + x^2y - xz^3$

(b)  $f(x, y, z) = x^4/(yz)$

(c)  $f(x, y, z) = (x^2 + y^3 + z^4)^6$

(d)  $f(x, y) = x^2 + e^{2y}$

(e)  $f(x, y) = y \ln x$

(f)  $f(x, y) = xy^2 - e^{xy}$