

ECON 4323/5301 Homework 2

Fall 2007

Due October 16, 2007 at 5:30 pm

1. Consider the function $f(x, y, z) = x^2 - y^2 + z^3$.
 - (a) If $x = 6$ and $y = 3$, find a value of z which satisfies $f(x, y, z) = 0$.
 - (b) Does this equation define z as an implicit function of x and y near $x = 6$ and $y = 3$?
 - (c) If so, compute

$$\frac{\partial z}{\partial x}(6, 3) \text{ and } \frac{\partial z}{\partial y}(6, 3).$$

2. For each of the functions defined on \mathbf{R}^3 , find the critical points and classify them as local maximum points, local minimum points, saddle points, or you cannot say what:
 - (a) $x^2 + 6xy + y^2 - 3yz + 4z^2 - 10x - 5y - 21z$,
 - (b) $(x^2 + 2y^2 + 3z^2)e^{-(x^2+y^2+z^2)}$.
3. An airline can treat business and pleasure travel as separate markets by demanding advance purchase and Saturday night stay-over for vacationers. Suppose the airline estimates a demand function of $Q_b = 16 - p_b$ for business travelers and a demand function of $Q_v = 10 - p_v$ for vacationers. The airline has a cost function for all travelers of $C(Q_t) = 10 + Q_t^2$, where Q_t is the total number of travelers. How much should the airline charge in each market to maximize its profit? Recall that profit is equal to total revenue minus total cost.
4. Find the maximum and minimum distance from the origin to the ellipse $x^2 + xy + y^2 = 3$. Recall that Euclidean distance from the origin to a point $(x, y) \in \mathbf{R}^2$ is $d = \sqrt{x^2 + y^2}$.
5. Find the general expression (in terms of all the parameters) for the commodity bundle (x_1, x_2) which maximizes the Cobb-Douglas utility function $U(x_1, x_2) = x_1^a k x_2^{1-a}$ on the budget set $p_1 x_1 + p_2 x_2 = w$. Assume $0 < a < 1$.
6. Find the point closest to the origin in \mathbf{R}^3 that is on both the planes $3x + y + z = 5$ and $x + y + z = 1$. Recall that Euclidean distance from the origin to a point $(x, y, z) \in \mathbf{R}^3$ is $d = \sqrt{x^2 + y^2 + z^2}$.

7. Maximize $f(x, y, z) = yz + xz$ subject to $y^2 + z^2 = 1$ and $xz = 3$.
8. If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, a firm produces $Q(x, y) = 50x^{1/2}y^2$ units of output.
- How should a budget of \$80,000 be allocated between labor and equipment to generate the largest possible output? (Don't bother with the second order conditions.)
 - Use the Envelope Theorem to estimate the change in maximum output if the budget is reduced by \$1000.
9. Use the Envelope Theorem to estimate the maximum and minimum distance from the origin to the ellipse $x^2 + xy + 0.9y^2 = 3$.
10. The owner of a small business has just purchased a personal computer, which she expects will serve her for the next 2 years. The owner has been told that she "must" buy a surge suppressor to provide protection for her new hardware against possible surges or variations in the electrical current that can damage the computer. The amount of the damage depends on the strength of the surge. There is a 1 percent chance of incurring damages that cost \$400, a 2 percent chance of incurring damages that cost \$200, and a 10 percent chance of incurring damages that cost \$100. If the owner makes decisions solely on the basis of expected value, what is the most she would be willing to pay for the surge suppressor?
11. Laser surgery to fix short-sightedness is becoming more popular. For some people, however, a second procedure is necessary. The following table lists the joint probabilities of needing a second procedure and whether the patient has a corrective lens with a factor (diopter) of minus 8 or less:

	Vision corrective factor:	
	more than minus 8	minus 8 or less
First procedure successful	0.66	0.15
Second procedure required	0.05	0.14

- Find the probability that a second procedure is required.
- Determine the probability that someone whose corrective lens factor is minus 8 or less does not require a second procedure.
- Define the random variable $X = 0$ if the first procedure is successful and $X = 1$ if a second procedure is required. Define the random variable $Y = 0$ if a patient has a corrective lens factor of minus 8 or less and $Y = 1$ if a patient has a corrective lens factor of more than

minus 8. Write down the marginal probability functions $f(x)$ and $g(y)$.

(d) Are X and Y independent? Explain.