

# ECON 4323/5301 Homework 3

Fall 2007

Due November 15, 2007 at 5:30 pm

1. Find the maximizer of  $f(x, y) = x^2 + y^2$  subject to the constraints that  $2x + y \leq 2$ ,  $x \geq 0$ , and  $y \geq 0$ .

2. Determine if the following functions are convex, concave, or neither:

(a)

$$f(x) = 3e^x + 5x^4 - \ln x$$

(b)

$$f(x, y) = -3x^2 + 2xy - y^2 + 3x - 4y + 1$$

(c)

$$f(x, y, z) = 3e^x + 5y^4 - \ln z$$

3. Determine which of the following functions are homogeneous. For the homogeneous functions, identify the degree of homogeneity.

(a)

$$f(x, y) = 3x^5y + 2x^2y^4 - 3x^3y^3$$

(b)

$$f(x, y) = 3x^5y + 2x^2y^4 - 3x^3y^4$$

(c)

$$f(x, y) = x^{1/2}y^{-1/2} + 3xy^{-1} + 7$$

4. Write down the degree-one homogenization of each of the following functions:

(a)

$$f(x) = e^x$$

(b)

$$f(x) = \ln x$$

(c)

$$f(x) = 5$$

(d)

$$f(x, y) = x^2 + y^3$$

5. Find the following integrals:

(a)

$$\int (x^3 + 2x - 3) dx$$

(b)

$$\int (x - 1)^2 dx$$

(c)

$$\int (x - 1)(x + 2) dx$$

(d)

$$\int (x + 2)^3 dx$$

(e)

$$\int (e^{3x} - e^{2x} + e^x) dx$$

(f)

$$\int \frac{x^3 - 3x + 4}{x} dx$$

6. Evaluate the following integrals:

(a)

$$\int_1^3 \frac{3x}{10} dx$$

(b)

$$\int_{-3}^{-1} x^2 dx$$

(c)

$$\int_0^1 \alpha e^{\beta x} dx, \beta \neq 0$$

(d)

$$\int_{-2}^{-1} \frac{1}{y} dy$$

7. Evaluate the following integrals:

(a)

$$\int_0^5 (x + x^2) dx$$

(b)

$$\int_{-2}^2 (e^x - e^{-x}) dx$$

(c)

$$\int_2^{10} \frac{dx}{x-1}$$

(d)

$$\int_0^1 2xe^{x^2} dx$$

(e)

$$\int_{-4}^4 (x-1)^3 dx$$

(f)

$$\int_1^2 (x^5 + x^{-5}) dx$$

8. Find:

(a)

$$\frac{d}{dt} \int_0^t x^2 dx$$

(b)

$$\frac{d}{dt} \int_t^3 e^{-x^2} dx$$

(c)

$$\frac{d}{dt} \int_{-t}^t \frac{1}{\sqrt{x^4 + 1}} dx$$

9. Evaluate the following:

(a)

$$\int_{-1}^1 x \ln(x + 2) dx$$

(b)

$$\int_0^2 x 2^x dx$$

(c)

$$\int_0^1 x^2 e^x dx$$

10. Find the following integrals by means of an appropriate substitution:

(a)

$$\int x(2x^2 + 3)^5 dx$$

(b)

$$\int x^2 e^{x^3+2} dx$$

(c)

$$\int \frac{2x - 1}{x^2 - x + 8} dx$$

(d)

$$\int x\sqrt{1+x}dx$$

(e)

$$\int \frac{x^3}{(1+x^2)^3}dx$$

(f)

$$\int x^5\sqrt{4-x^3}dx$$

11. Find an expression for  $F'(x)$  when

(a)

$$F(x) = \int_1^2 \frac{e^{xt}}{t} dt, x \neq 0$$

(b)

$$F(x) = \int_1^e \ln(xt) dt, x > 0$$

(c)

$$F(x) = \int_0^1 \frac{e^{-t}}{1+xt} dt, x > -1$$

(d)

$$F(x) = \int_3^8 \frac{t^2}{(1-xt)^2} dt, x > 1/3$$

12. Use Leibniz's Formula to find an expression for  $F'(x)$  when

(a)

$$F(x) = \int_0^{2x} t^3 dt$$

(b)

$$F(x) = \int_0^x (x^2 + t^3)^2 dt$$