ECON 4323/5301 Homework 4

Fall 2007

Due December 6, 2007 at 5:30 pm

1. Find the general solutions to the following linear differential equations:

   (a)

   $$\frac{dy}{dt} + y = 10,$$

   (b)

   $$\frac{dy}{dt} - 3y = 27,$$

   (c)

   $$4\frac{dy}{dt} + 5y = 100.$$

2. Find the general solutions to the following differential equations:

   (a)

   $$t\frac{dy}{dt} + 2y + t = 0, t \neq 0,$$

   (b)

   $$\frac{dy}{dt} - \frac{1}{t}y = t, t > 0,$$

   (c)

   $$\frac{dy}{dt} - \frac{1}{t^2 - 1}y = t, t > 1,$$

   (d)

   $$\frac{dy}{dt} - \frac{2}{t}y + \frac{2a^2}{t} = 0, t > 0.$$

3. Solve the differential equation

$$1 + \left(2 + \frac{t}{y}\right)\frac{dy}{dt} = 0, t > 0, y > 0.$$

4. Solve the following Bernoulli equations assuming $t > 0$, $y > 0$:

   (a)

$$t\frac{dy}{dt} + 2y = ty^2,$$

   (b)

$$\frac{dy}{dt} = 4y + 2e^t\sqrt{y},$$

   (c)

$$t\frac{dy}{dt} + y = y^2 \ln t.$$

5. An economic growth model by Haavelmo (1954) leads to the differential equation

$$\frac{dK}{dt} = \gamma_1 bK^\alpha + \gamma_2 K,$$

   where γ_1, γ_2, b, and α are positive constants, $\alpha \neq 1$, and $K = K(t)$ is the unknown function. The equation is separable, but solve it as a Bernoulli equation.

6. A study of the optimal exhaustion of a natural resource uses the equation

$$\frac{d^2y}{dt^2} - \frac{2-\alpha}{1-\alpha}a\frac{dy}{dt} + \frac{a^2}{1-\alpha}y = 0,$$

   where $\alpha \neq 0$, $\alpha \neq 1$, and $a \neq 0$. Prove that $u_1 = e^{at}$ and $u_2 = e^{at/(1-\alpha)}$ are both solutions. What is the general solution?

7. Find the general solutions of the following equations:

   (a)

$$\frac{d^2y}{dt^2} - 3y = 0,$$

   (b)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 0,$$

(c)

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 8,$$

(d)

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{5t},$$

(e)

$$\frac{d^2y}{dt^2} - y = e^{-t},$$

(f)

$$3\frac{d^2y}{dt^2} - 30\frac{dy}{dt} + 75y = 2t + 1.$$