Homework 1 Solutions

ECON 5332 Government, Taxes, and Business Strategy
Spring 2008

January 22, 2008

1. Consider an income guarantee program with an income guarantee of \$3000 and a benefit reduction rate of 50 percent. A person can work up to 2000 hours per year for \$6 per hour. Alice, Bob, Calvin, and Deborah work for 100, 333 1/3, 400, and 600 hours, respectively, under this program.

   The government is considering altering the program to improve work incentives. Its proposal has two pieces. First, it will lower the guarantee to \$2000. Second, it will not reduce the benefits for the first \$3000 earned by workers. After this, it will reduce benefits at a reduction rate of 50 percent.

   The two goods Alice, Bob, Calvin, and Deborah consume are leisure (L) and a consumption good (C) that represents the aggregation of everything else they like other than leisure. Without loss of generality, let the price of the consumption good be equal to 1. We will then refer to the consumption good as the numeraire.

   (a) Draw the budget constraint facing any worker under the original program.

   See Figure 1a. The original budget constraint is the blue line connecting points a, c, d, and e.

   (b) Draw the budget constraint facing any worker under the proposed new program.

   See Figure 1a. The budget constraint under the proposed program is the black line connecting points a, b, f, g, and e.

   The two budget constraints are the same between points a and b, i.e. between 0 and 833 hours of leisure consumption. The two budget constraints intersect again at point h, which is the bundle containing 1666 2/3 hours of leisure consumption and \$4000 worth of other consumption.

   (c) Which of the four workers do you expect to work more under the new program? Who do you expect to work less? Are there any workers for whom you cannot tell if they will work more or less?

   Workers working fewer than 500 hours see their hourly wage effectively doubled under the plan. The substitution effect therefore tends to make Alice, Bob, and Calvin all work more. One can calculate that the two budget constraints cross at 333 1/3 hours of work, or 1,666 2/3 hours of leisure. The income effect is thus different for these three workers. Alice was working less than 333 1/3 hours under the old policy, so the policy change effectively makes her poorer. She consumes less of all normal goods, including leisure, so this also makes her work more. We can unambiguously conclude that she will work more. Bob was working exactly 333 1/3 hours, so he feels no income effect. We can conclude from the substitution effect alone that he too will work more. Calvin was working more than 333 1/3 hours before, so this policy change effectively makes him richer. He will therefore tend to work less due to the income effect. We cannot tell if the substitution effect or the income effect is stronger, so we cannot tell if Calvin will work more or less. Finally, Deborah was working 600 hours before. Under both policies, the effective wage of someone working this many hours is \$3/hr (since 50 percent of income is offset by reduced benefits). There is no substitution effect for her. As the graph shows, however, she experiences an increase in income. We conclude that she will work less. See Figures 1b - 1f.

2. Consider a perfectly competitive market with demand equal to $Q^d = 1200 - 10P$ and supply equal to $Q^s = 20P$.

   (a) How much do consumers want to purchase if $P = \$10$?

   At $P = \$10$, quantity demanded is 1100.

   (b) How much do producers want to sell if $P = \$10$?

   At $P = \$10$, quantity supplied is 200.

   (c) What is the equilibrium price and quantity in the market?

   The equilibrium price is the price at which $Q^d = Q^s$. Substituting, we see that

   $$1200 - 10P = 20P.$$

   Solving for P gives us the equilibrium price, $P^* = \$40$. At this price $Q^d = Q^s = Q^* = 800$.

   (d) What is the most consumers are willing to pay for the 100th unit?

   The willingness-to-pay function is the inverse of the demand function, so $WTP = 120 - (1/10)Q$. It follows that consumers are willing to to pay at most \$110 for the 100th unit.

   (e) What is the marginal cost of the 400th unit?

   The marginal cost function is the inverse of the supply function, so $MC = (1/20)Q$. It follows that the marginal cost of the 400th unit is \$20.

   (f) What is the value of consumer surplus, producer surplus, and total surplus?

   See Figure 2a. Consumer surplus is equal to $(1/2)(800)(120 - 40) = \$32,000$. Producer surplus is equal to $(1/2)(800)(40) = \$16,000$. Total surplus is equal to \$48,000.

   (g) Suppose the government imposes a \$10 per unit subsidy on the production of the good. What is consumer surplus after the subsidy is implemented? What is producer surplus after the subsidy is implemented? What is total surplus? Why is there a deadweight loss associated with this subsidy and what is the size of this loss?

The subsidy reduces costs by \$10 per unit, so the new marginal cost function is $MC = (1/20)Q - 10$. It follows that the new supply curve is $Q^s = 20P + 200 = 20(P+10)$. See Figure 2b. A calculation similar to the one in part (c) shows that the new equilibrium price is $P^* = \$33.33$ and the new equilibrium quantity is $Q^* = 866.67$. We can see from Figure 3b that consumer surplus after the subsidy is $(1/2)(866.67)(120 - 33.33) \approx \$37,555.56$. Since producers get a \$10 subsidy in addition to the price consumers pay, producer surplus is $(1/2)*(866.67)*(33.33+10) \approx \$18,777.78$. The total subsidy is equal to $(10)*(866.67) \approx \$8666.7$. It follows that total surplus is approximately equal to $\$37,555.56 + \$18,777.78 - \$8666.7 = \$47,666.6$. Since total surplus is less than it was without the subsidy, there is a deadweight loss equal to the reduction in total surplus. The deadweight loss is equal to the area of the green triangle in Figure 3b, which is $(1/2)(866.67 - 800)(10) = 333.3$. This is approximately equal to the difference in total surplus without and with the subsidy, which is $\$48,000 - \$47,666,666.6 = \$333.4$. The deadweight loss exists because the marginal cost of producing units 801 and above exceeds the marginal benefit to consumers of purchasing them.

3. Dexy has \$3000 to spend on entertainment this year. The price of a day trip (T) is \$40 and the price of dinner and a movie (M) is \$20. Suppose Dexy's utility function is $U(T,M) = T^{1/3}M^{2/3}$. It follows that the marginal utility of a day trip is $MU_T = (1/3)(M/T)^{2/3}$ and the marginal utility of dinner and a movie is $MU_M = (2/3)(T/M)^{1/3}$.

   (a) What is the utility maximizing combination of day trips and dinner-movie dates?

   The optimal bundle will satisfy the properties that the marginal rate of substitution is equal to the price ratio and that the budget constraint is satisfied with equality. The marginal rate of substitution is

$$MRS = \frac{MU_T}{MU_M} = \frac{(2/3)(T/M)^{1/3}}{(1/3)(M/T)^{2/3}} = \frac{M}{2T}.$$

   Here is a good way to figure out what the marginal rate of substitution is telling us. The marginal utility of day trips (the numerator) is measured in utils per trip,

$$\frac{utils}{trip},$$

   and the marginal utility of movies (the denominator) is measured in utils per movie,

$$\frac{utils}{movie}.$$

   The units for the marginal rate of substitution must be

$$\frac{utils/trip}{utils/movie} = \frac{movies}{trip}.$$

Thus, the marginal rate of substitution tells us how many movies Dexy is willing to give up to take an additional day trip.

The price ratio is

$$\frac{P_T}{P_M} = \frac{40}{20} = 2.$$

The price ratio tells us how many movies Dexy actually has to give up to take an additional trip. At the optimal bundle, the number of movies Dexy is willing to trade for an additional trip must be equal to the number of movies she does trade for an additional trip. In other words,

$$\frac{M}{2T} = 2, \text{ or } M = 4T.$$

The budget constraint is $20M + 40T = 3000$. Substituting $4T$ for M and solving for T we see that $T^* = 25$ and $M^* = 100$.

(b) Suppose the price of day trips increases to \$50 per trip. How will this change the optimal consumption bundle?

The price ratio is now

$$\frac{P_T}{P_M} = \frac{50}{20} = 2.5.$$

It follows that

$$\frac{M}{2T} = 2.5, \text{ or } M = 5T.$$

The budget constraint is now $20M + 50T = 3000$. Substituting for M we get that $T^* = 20$ and $M^* = 100$.

You could also find the marginal rate of substitution of trips for movies, i.e. the number of day trips Dexy is willing to give up to have an additional dinner-movie date. In this case, the marginal rate of substitution is

$$MRS = \frac{MU_M}{MU_T} = \frac{2T}{M}.$$

If you do it this way, just be sure that your price ratio is measuring the actual number of day trips Dexy has to trade to get an additional dinner-movie date, which is

$$\frac{P_M}{P_T}.$$

Just remember is that the same good is in the numerator of both the marginal rate of substitution and the price ratio, and similarly for the denominator.

Figure 1a Budget constraints under the original and proposed income guarantee programs

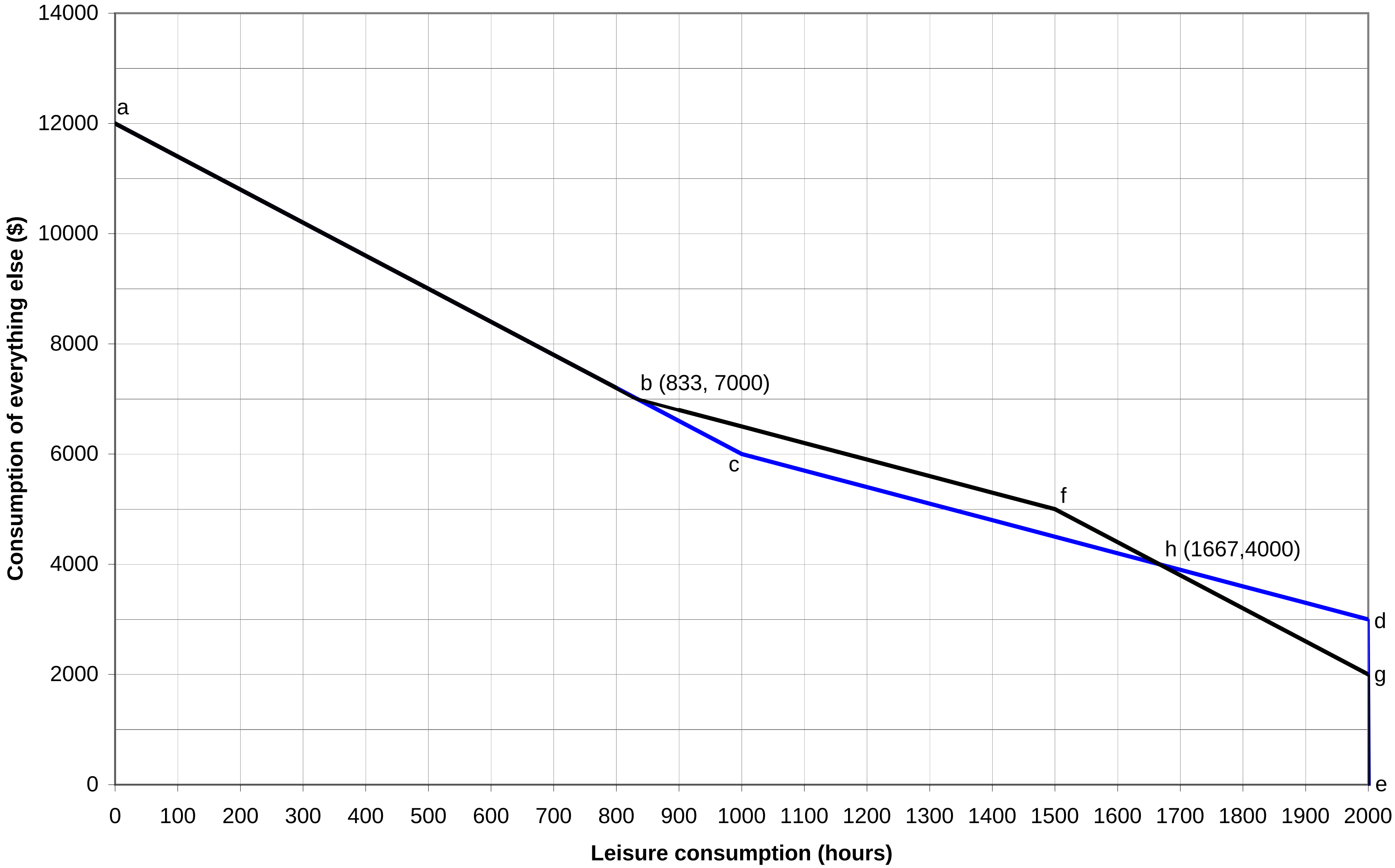

Figure 1b Consumption-leisure choices for Alice

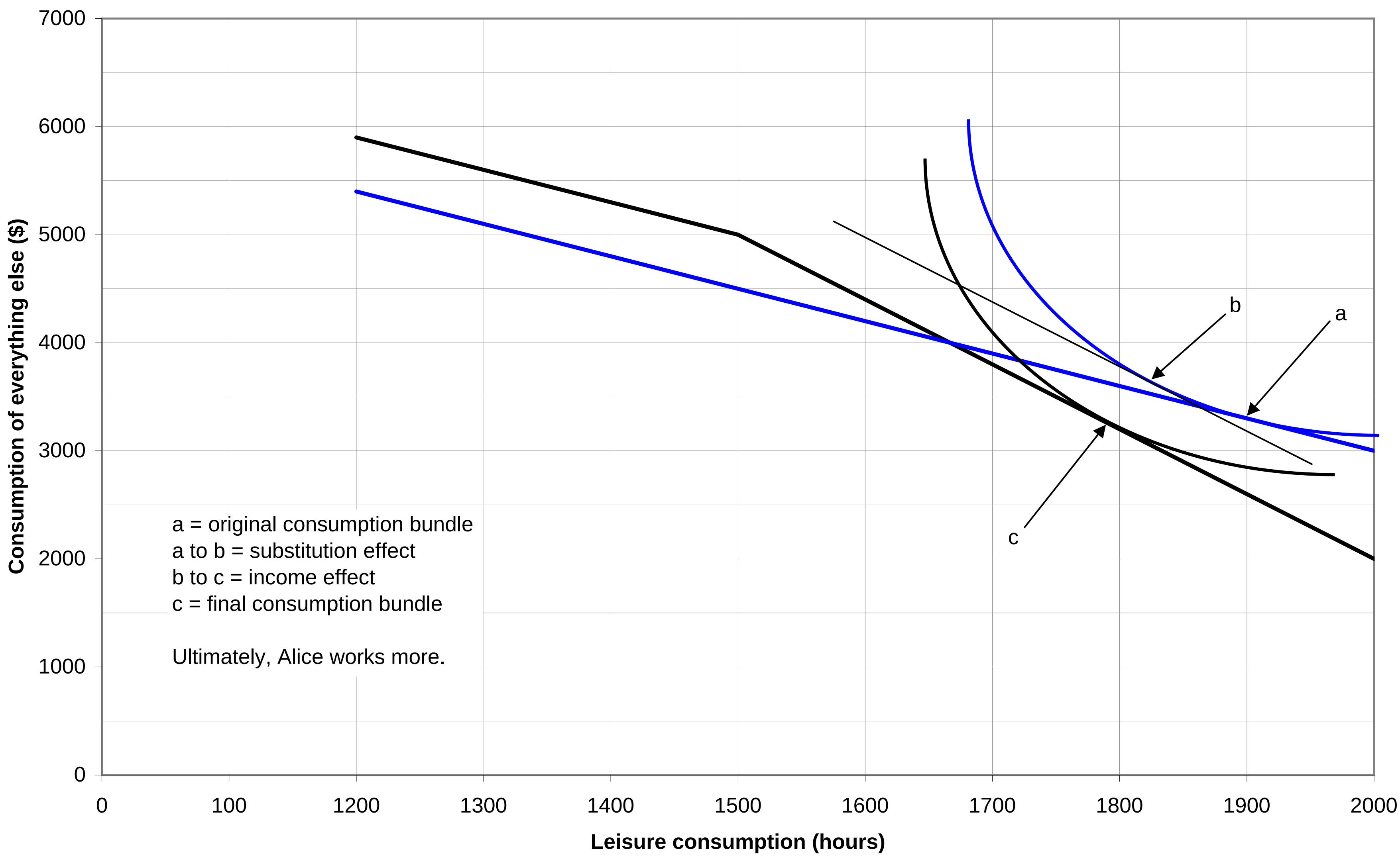

Figure 1c Consumption-leisure choices for Bob

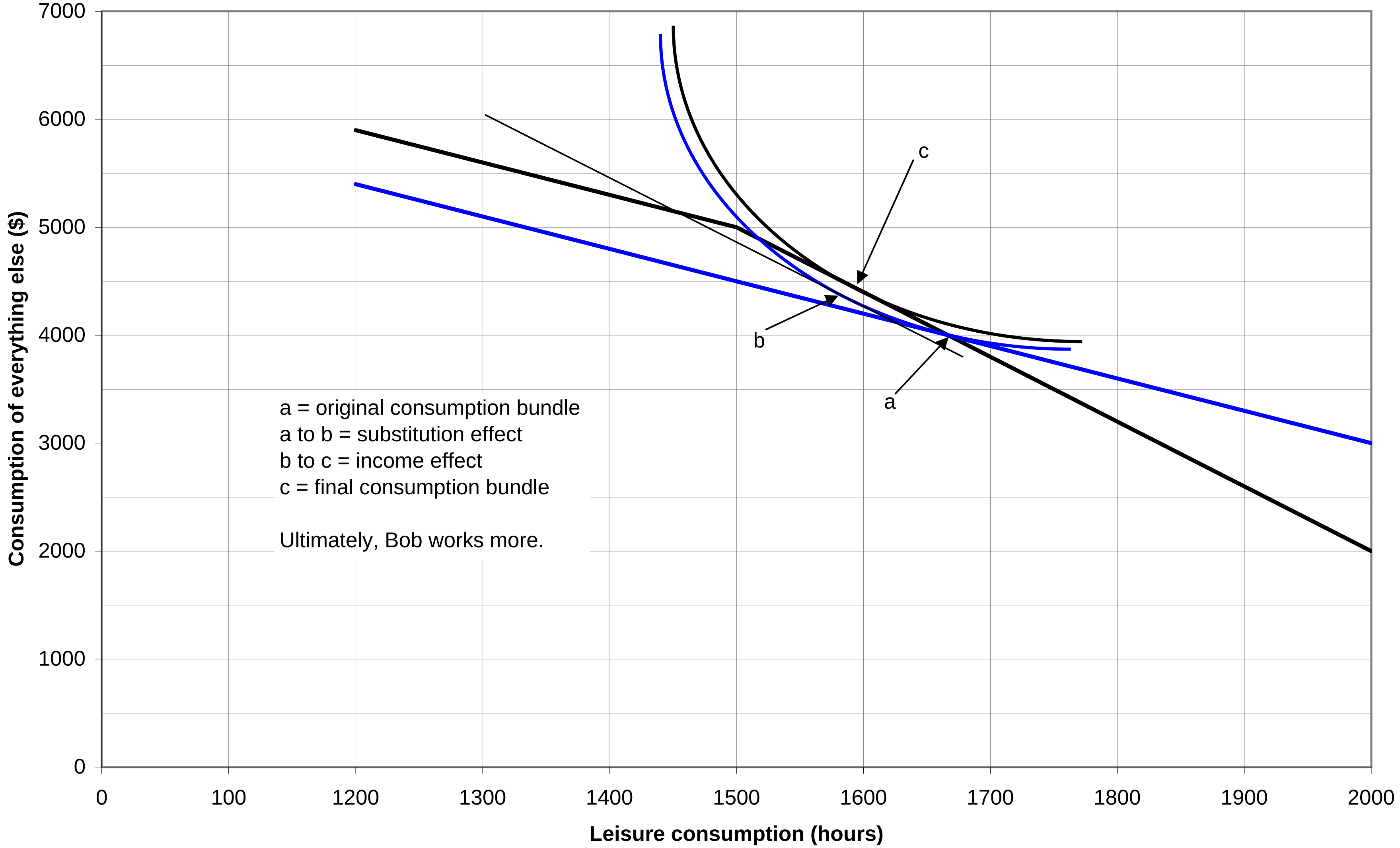

Figure 1d Consumption-leisure choices for Calvin

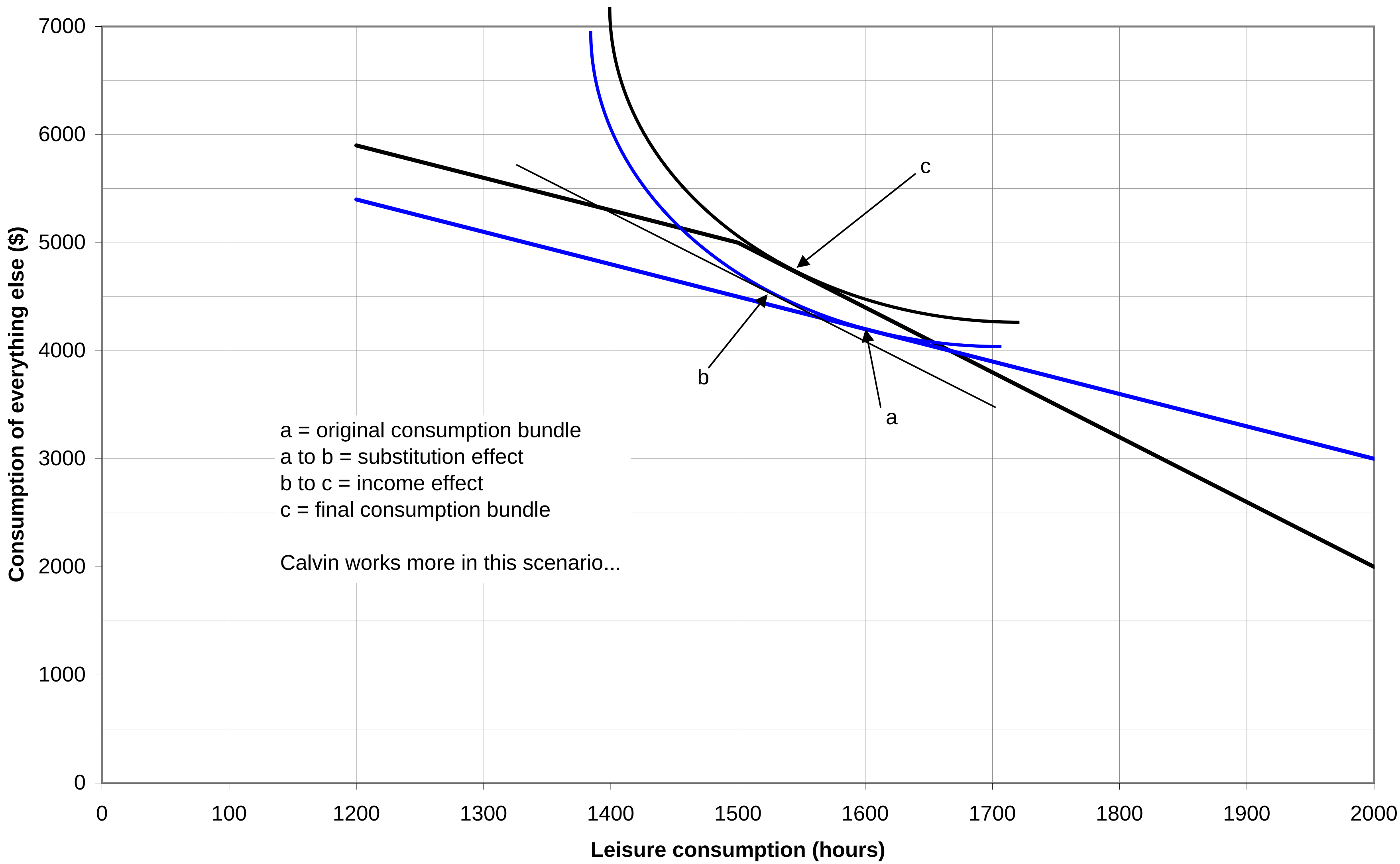

Figure 1e Consumption-leisure choices for Calvin

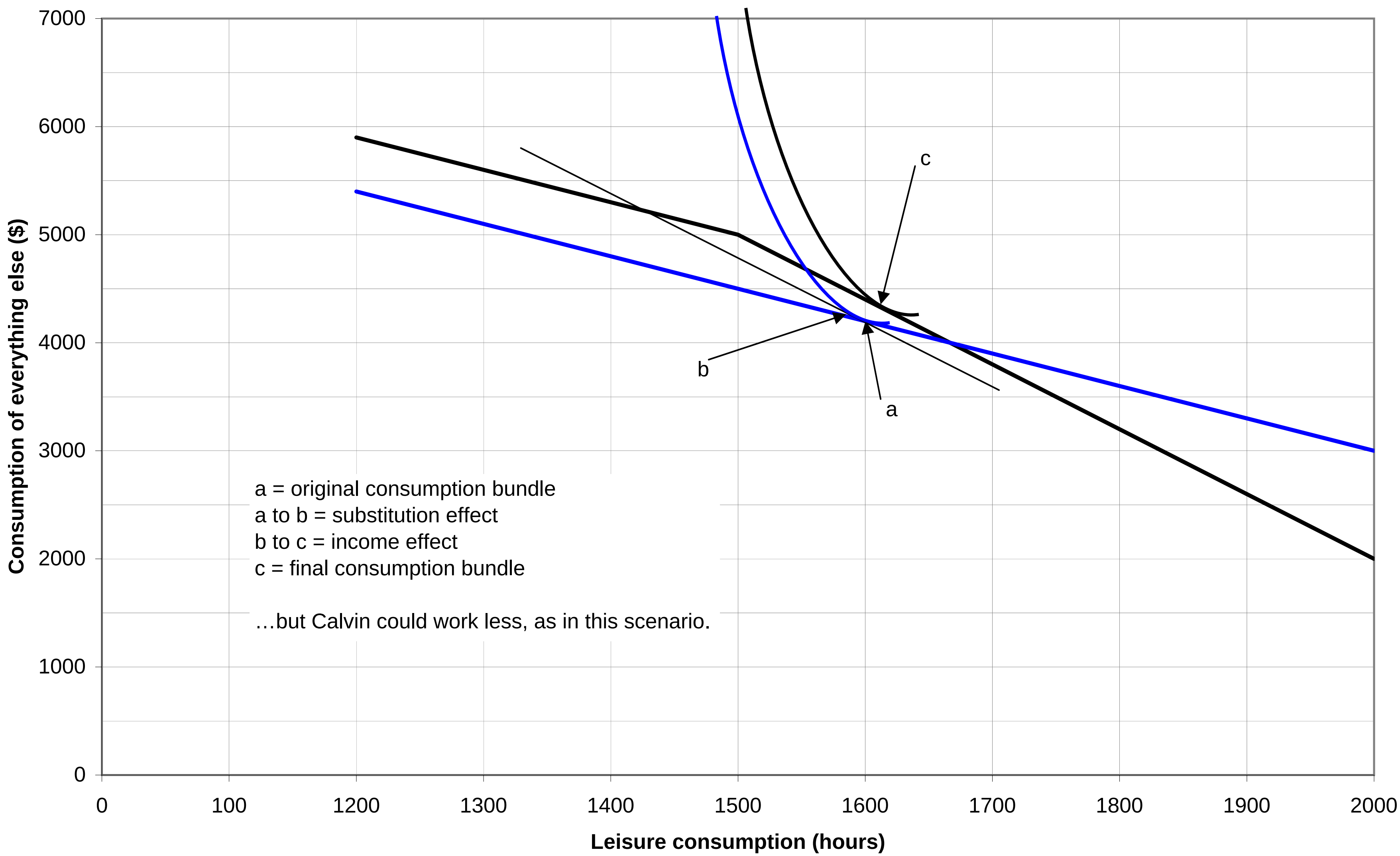

Figure 1f Consumption-leisure choices for Deborah

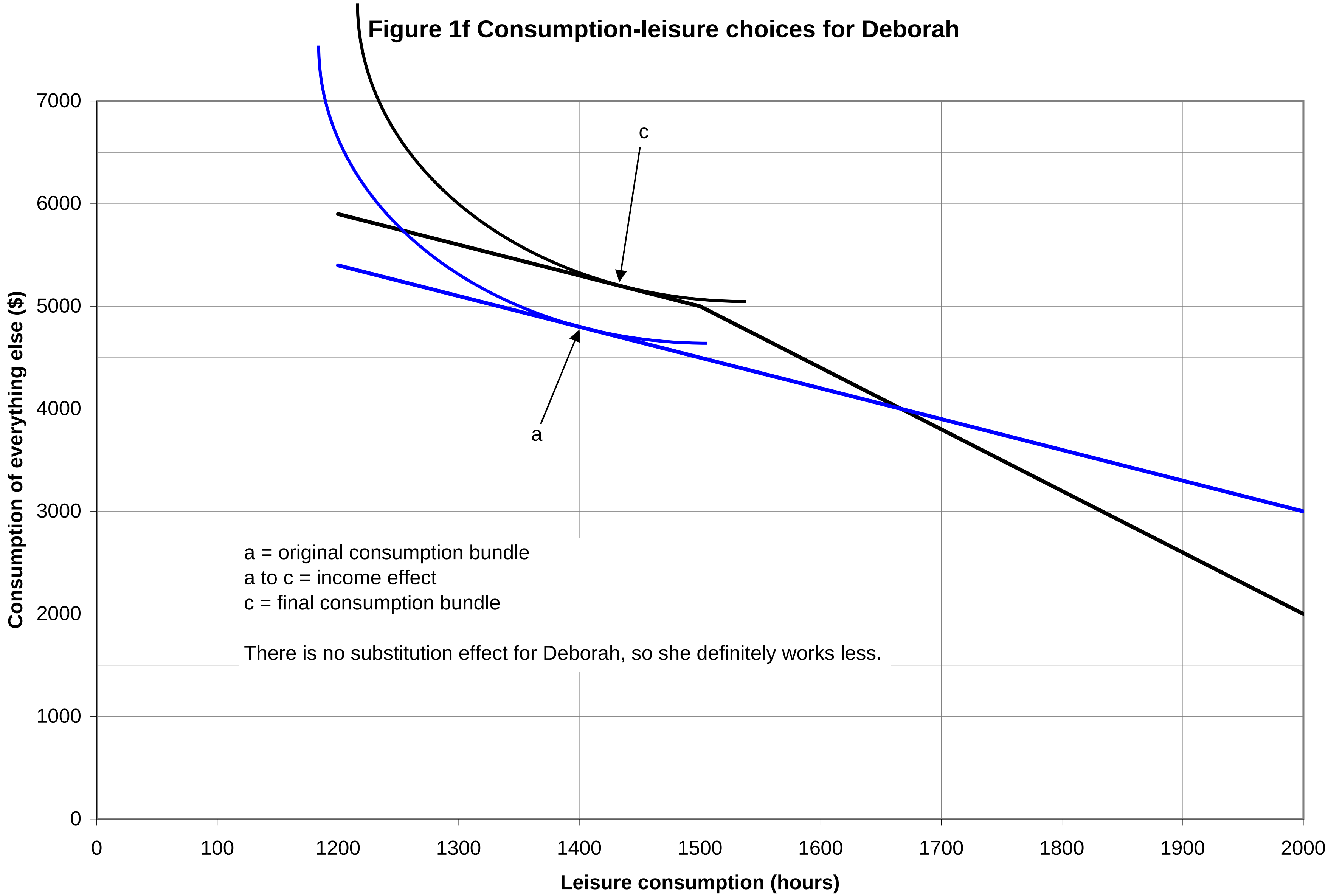

Figure 2a Consumer and producer surplus

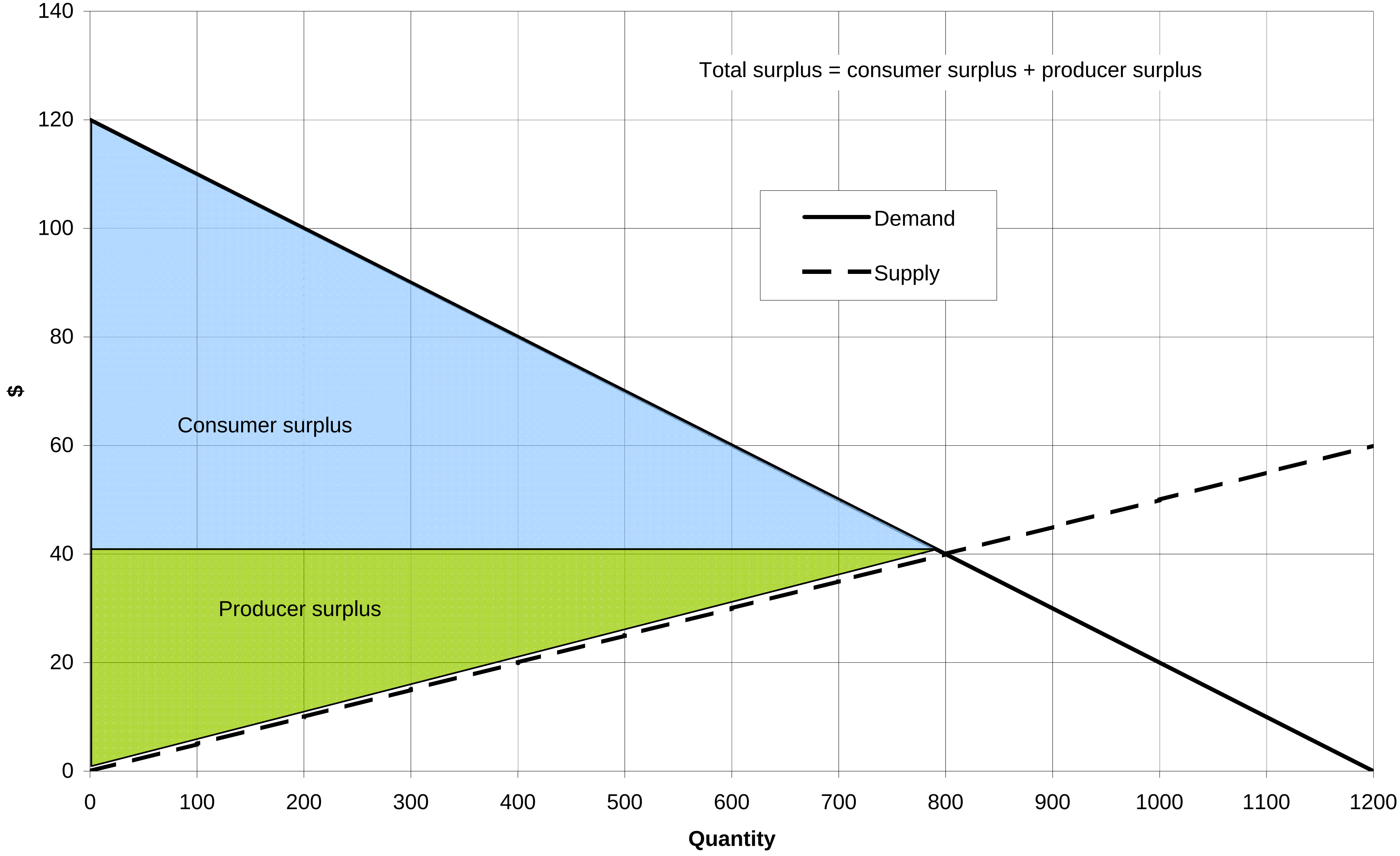

Figure 2b Consumer and producer surplus with the subsidy