

Homework 2 Solutions

ECON 5332 Government, Taxes, and Business Strategy
Spring 2008

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1. Warrenia has two regions. In Oliviland, the marginal benefit associated with pollution cleanup is $MB = 300 - 10Q$, while in Linneland, the marginal benefit associated with pollution cleanup is $MB = 200 - 4Q$. Suppose the marginal cost of cleanup is constant at \$12 per unit. What is the optimal level of pollution cleanup in each of the two regions?

The optimal level of cleanup will occur when the marginal benefit just equals the marginal cost. In Oliviland, the marginal benefit is $300 - 10Q$; marginal cost is 12. Therefore, the equation to solve for Oliviland is $300 - 10Q = 12$, or $288 = 10Q$. The optimal level in Oliviland is equal to 28.8. For Linneland, the marginal benefit is $200 - 4Q$. Setting the benefit equal to 12 yields $200 - 4Q = 12$, or $188 = 4Q$. The optimal level in Linneland is equal to 47.

2. The marginal private benefit associated with a product's consumption is $MPB = 360 - 4Q$ and the marginal private cost associated with its production is $MPC = 6Q$. Furthermore, the marginal external damage associated with this good's production is $MEC = 2Q$. To correct the externality, the government decides to impose a tax of T per unit sold. What tax T should it set to achieve the social optimum?

Find the social optimum by setting $MSB = MPB = PMC + MEC = MSC$:

$$360 - 4Q = 8Q, \text{ or } Q^* = 30.$$

The marginal external cost at $Q^* = 30$ is $MEC = 2(30) = 60$. It follows that a Pigouvian tax of $T = 60$ will achieve the social optimum.

3. Suppose the demand for a product is $Q = 1200 - 4P$ and supply is $Q = -200 + 2P$. Furthermore, suppose that the marginal external damages of this product is \$8 per unit. How many more units of this product will the market produce relative to the social optimum? Calculate the deadweight loss associated with this externality.

To answer this question, first calculate what the free market would do by setting quantity demanded equal to quantity supplied:

$$1,200 - 4P = -200 + 2P.$$

It follows that $1,400 = 6P$, or $P^* \approx 233.33$. This implies that $Q^* = 1,200 - 4(233.33) \approx 266.67$.

The socially optimal quantity is the quantity for which the marginal social benefit equals the marginal social cost. Without loss of generality, we will include the external damages in the calculation of marginal social cost. The marginal private cost function is the inverse of the supply function, so

$$MPC = (1/2)Q + 100.$$

The marginal external cost function is simple,

$$MEC = 8.$$

It follows that the marginal social cost function is

$$MSC = MPC + MEC = (1/2)Q + 100 + 8 = (1/2)Q + 108.$$

Since we have included the externality in the calculation of marginal social cost, marginal social benefit is just equal to marginal private benefit. The marginal private benefit function is the inverse of the demand function, so

$$MPB = 300 - (1/4)Q.$$

It follows that the marginal social benefit function in this case is

$$MSB = 300 - (1/4)Q.$$

To find the socially optimal quantity, we set $MSB(Q) = MSC(Q)$ and solve for Q :

$$300 - (1/4)Q = (1/2)Q + 108,$$

or $Q^\circ = 256$.

Since $Q^* \approx 266.67$ and $Q^\circ = 256$, the market provides about 10.67 units more than the social optimum.

The deadweight loss is the amount by which marginal social benefit exceeds marginal social cost on a unit, summed over all units the market produces in excess of the social optimum:

$$DWL = (1/2)(266.67 - 256)(8) \approx 42.68.$$