Homework 4 Solutions

ECON 5332 Government, Taxes, and Business Strategy
Spring 2008

Due Tuesday, February 12, at 7:00 pm

1. Bills demand for hamburgers (a private good) is $Q = 20 - 2P$ and Teds demand is $Q = 10 - P$.

   (a) Write down an equation for the marginal social benefit of the consumption of hamburgers.

   The marginal social benefit from the Qth hamburger is the willingness of society to pay for the hamburger. To compute it, we first find the market demand curve for hamburgers by summing individual demands horizontally: $Q = (20 - 2P) + (10 - 3P) = 30 - 3P$. Inverting this gives $P = 10 - Q/3$. The willingness of society to pay for the Qth hamburger is thus $10 - Q/3$.

   (b) Now suppose that hamburgers are a public good. Write down an equation for the marginal social benefit of hamburger consumption.

   For public goods, we do not add quantities horizontally; we add marginal private benefits vertically. For Bill, $Q = 20 - 2P$; solving for P yields $P = 10 - 0.5Q$, so $MPB_B = 10 - 0.5Q$. For Ted, $Q = 10 - P$, so $P = 10 - Q$ and thus $MPB_T = 10 - Q$. Summing vertically, $MSB = MPB_B + MPB_T = (10 - 0.5Q) + (10 - Q) = 20 - 1.5Q$.

2. The town of Springfield has two residents: Homer and Bart. The town currently funds its fire department solely from the individual contributions of these residents. Each of the two residents has a utility function over private goods (X) and total fire-fighters (F) of the form $U = 4\ln(X) + 2\ln(F)$. It follows that the marginal utility of private goods is $MU_X = 4/X$ and the marginal utility of fire-fighters is $MU_F = 2/F$. The total provision of fire-fighters hired, F, is the sum of the number hired by each of the two persons: $F = F^H + F^B$. Homer and Bart both have income of \$100, and the price of both the private good and a fire-fighter is \$1. Thus, they are limited to providing between 0 and 100 fire-fighters.

   (a) How many fire-fighters are hired if the government does not intervene? How many are paid for by Homer? By Bart?

   Each resident optimizes his own utility, so each will choose the number of fire-fighters that does so, taking into consideration the number of fire-fighters the other resident contributes. Let's start with Bart. Bart's marginal rate of substitution of X for F is

$$MRS^B = \frac{MU_F}{MU_X} = \frac{2/F}{4/X^B} = \frac{X^B}{2F}.$$

The price ratio is 1. Setting $MRS^B = 1$, we have $X = 2F$. Bart's budget constraint is $X^B + F^B = 100$, so $X^B = 100 - F^B$ (all income not spent on fire-fighters is spent on private goods). The quantity of public good Bart enjoys is $F^B + F^H$, because public goods provided by either Bart or Homer are consumed by both. Substituting, we have $100 - F^B = 2(F^B + F^H)$. It follows that the number of fire-fighters Bart provides is given by $F^B = (100 - 2F^H)/3$.

Symmetry implies that the number of fire-fighters Homer provides is given by $F^H = (100 - 2F^B)/3$.

In equilibrium, neither person wants to change their behavior (i.e. the number of fire-fighters he hires). We can solve for Bart's equilibrium number of fire-fighters by substituting for F^H in his reaction function:

$$F^B = \frac{100}{3} - \frac{2}{3}F^H = \frac{100}{3} - \frac{2}{3}\left(\frac{100}{3} - \frac{2}{3}F^B\right) = \frac{100}{9} + \frac{4}{9}F^B.$$

It follows that

$$\frac{5}{9}F^B = \frac{100}{9} \Rightarrow F^B = 20.$$

It follows that $F^H = 20$ as well. The total number of fire-fighters is $F^* = 40$.

(b) What is the socially optimal number of fire-fighters? If your answer differs from your answer to part (a), why?

The socially optimal number of fire-fighters is determined by first summing residents' marginal private benefits of fire-fighters to determine the marginal social benefit of fire-fighters, and then finding the fire-fighter whose marginal social benefit equals marginal (social) cost. Recall that the marginal private benefit of a fire-fighter, measured in terms of private good, is given by the marginal rate of substitution of private good for fire-fighters (placing the marginal utility for the public good in the numerator and for the private good in the denominator). Thus, the marginal social benefit of fire-fighters is

$$\begin{aligned} MSB &= MPB^B + MPB^H \\ &= \frac{MU_F^B}{MU_X^B} + \frac{MU_F^H}{MU_X^H} \\ &= \frac{X^B}{2F} + \frac{X^H}{2F} \\ &= \frac{100 - F^B}{2(F^B + F^H)} + \frac{100 - F^H}{2(F^B + F^H)} \\ &= \frac{200 - (F^B + F^H)}{2(F^B + F^H)} \\ &= \frac{200 - F}{2F}. \end{aligned}$$

The price of a unit of private good equals the price of a fire-fighter, \$1, so the marginal social cost of fire-fighters, measured in terms of private good forgone, is 1. Thus, the socially optimal number of fire-fighters satisfies

$$\begin{aligned} MSB &= MSC \\ \frac{200-F}{2F} &= 1 \\ 200-F &= 2F \\ 200 &= 3F \\ F^{\circ} &= 200/3 \approx 66.67. \end{aligned}$$

Intuitively, in the computation in part (a), we set the marginal private benefit of the last fire-fighter to each resident equal to the marginal private cost to that resident. In part (b), we set the sum of the marginal private benefits of the last fire-fighter - the marginal social benefit of the fire-fighter - equal to the marginal cost for either resident. Since the marginal social benefit of fire-fighters exceeds the individual marginal benefit of that fire-fighter, the socially optimal number of fire-fighters is greater than the number individuals would hire if they were acting independently.

3. The town of Musicville has two residents: Bach and Mozart. The town currently funds its free outdoor concert series solely from the individual contributions of these residents. Each of the two residents has a utility function over private goods (X) and total concerts (C) of the form $U = 3\ln(X) + \ln(C)$. It follows that the marginal utility of private good is $MU_X = 3/X$ and the marginal utility of concerts is $MU_C = 1/C$. The total number of concerts given, C, is the sum of the number paid for by each of the two persons: $C = C^B + C^M$. Bach and Mozart both have income of 70, and the price of both the private good and a concert is \$1. Thus, they are limited to providing between 0 and 70 concerts.

   (a) How many concerts are given if the government does not intervene?

   Using the same method we use in part (a) of the previous problem, we find that $C^B = C^M = 10$ and the total number of concerts is $C = 20$.

   (b) Suppose the government is not happy with the private equilibrium and decides to provide 10 concerts in addition to what Bach and Mozart may choose to provide on their own. It taxes Bach and Mozart equally to pay for the new concerts. What is the new total number of concerts? How does your answer compare to your answer to part (a)? Have we achieved the social optimum? Why or why not?

   At a price of 1, providing 10 concerts will cost 10. Thus, paying for the new concerts funded by the government will require a tax of 5 on each of the two residents. After-tax income for each has declined to 65. Using the same method we used in part (a) and the fact that the total number of concerts provided is $C^B + C^M + 10$, we find that each resident will provide 5 concerts. It follows that a total of $5 + 5 + 10 = 20$ concerts are provided.

Using the same method used in part (b) of the previous problem, we find that the socially optimal number of concerts is $C^{\circ} = 35$. Note that income is equal to 70 for the social optimum calculation.

Government provision of concerts is not efficient because there is complete crowding out of private concert provision.

(c) Suppose that an anonymous benefactor pays for 10 concerts instead of the government. What is the new total number of concerts? Is this the same level of provision as in part (b)? Why or why not?

In this case, income does not decline, but total provision is $C^B + C^M + 10$. In this case, each resident will provide about 5.7 concerts and the total number of concerts will increase to about 21.4. There is incomplete crowding out because the donated concerts increase the wealth of each resident and allow each to purchase more concerts.

4. In your own words, state the two hypotheses that Brunner (1998) tests and explain the intuition motivating them.

One hypothesis is that average listener contributions to a public radio station decrease with the total number of listeners. The second hypothesis is that contributions to a public radio station per contributing listener decrease with the total number of listeners. Both hypotheses are motivated by the intuition that a listener's incentive to free ride or "easy ride" on other listeners' contributions increases with the total number of listeners. Free riding becomes more attractive as the number of listeners increases because it is more difficult to detect and punish.

5. According to Brunner (1998), what are the arguments of an individual's utility function in the impure altruist model? What two special cases does this model include and what are the arguments of an individual's utility function in each case?

In the impure altruist model, an individual's utility is a function of his or her private good consumption, his or her contribution to the public good, and the total quantity of public good provided. This model includes the pure altruist model and the pure egoist model as special cases. An individual's utility is a function of his or her private good consumption and total public goods provision (but not his or her contribution to the public good) in the pure altruist model. An individual's utility is a function of his or her private good consumption and his or her contribution to the public good (but not total public goods provision) in the pure egoist model.

6. Based on the intuition provided by Brunner (1998), are individuals motivated by altruism more or less likely to contribute to the provision of a public good than individuals motivated by egoism, all else being equal? Explain.

In the pure altruist model, an individual cares only about total public goods provision, so other people's contributions to the public good are perfect substitutes to his or her own contribution. In the pure egoist model, an individual cares only about his or her own contribution, so other people's contributions have no affect on his or her utility. Since pure altruists can essentially substitute other people's contributions for their own, they are more likely to free ride than pure egoists. Pure egoists are more likely to contribute to provision of the public good because, for them, there is no substitute for their own contribution.

7. With which model does Brunner (1998) say his results are consistent and why?

   Brunner (1998) states that his results are consistent with the impure altruist model, because the number of listeners who contribute to a public radio station decreases with the total number of listeners, but not as much as the pure altruist model predicts. One possible explanation for the smaller than expected decline in the number of contributing listeners is that people get a "warm glow" from contributing, so there is a potential role for egoism.

References

Eric J. Brunner. Free riders or easy riders?: An examination of the voluntary provision of public radio. *Public Choice*, 97:587–604, 1998. Available on EconLit.