MAE 4310 SOLUTION TO HW # 2

Problem 1)

$$\dddot{y}(t) + 8\dot{y}(t) + 25y(t) = 2\dot{u}(t) + u(t)$$

$y(0) = 1$, $\dot{y}(0) = 0$ and $\ddot{y}(0) = 0$

$$\left[s^3Y(s) - s^2y(0) - sy'(0) - y''(0)\right] + 8\left[sY(s) - y(0)\right]$$

$$+ 25Y(s) = 2s\,U(s) + U(s)$$

$$s^3Y(s) - s^2 + 8sY(s) - 8 + 25Y(s) = (2s+1)U(s)$$

$$Y(s)\left[s^3 + 8s + 25\right] = (2s+1)U(s) + s^2 + 8$$

$$Y(s) = \frac{(2s+1)U(s) + s^2 + 8}{s^3 + 8s + 25}$$

(i)

$$u(t) = 3 \times 1(t) \xrightarrow{\mathcal{L}} U(s) = 3/s$$

$$Y(s) = \frac{(2s+1).\ 3/s + s^2 + 8}{s^3 + 8s + 25}.$$

$$Y(s) = \frac{6s+3+s^3+8s}{s(s^3+8s+25)}$$

$$Y(s) = \frac{s^3+14s+3}{s(s^3+8s+25)}$$

Partial fraction expansion using Matlab

[R, P, k] = residue ([1 0 14 3], [1 0 8 25 0])

$$Y(s) = \frac{0.0338-0.239i}{s-1.0246-3.339i} + \frac{0.0338+0.239i}{s-1.0246+3.339i} + \frac{0.8125}{s+2.0493} + \frac{0.12}{s}$$

$$Y(t) = (0.0338-0.239i)\,e^{(1.0246+3.339i)t} + (0.0338+0.239i)\,e^{(1.0246-3.339i)t}$$
$$+ 0.8125\,e^{-2.0493t} + 0.12$$

$$\boxed{(a+bi)e^{(\alpha-\beta i)t} + (a-bi)e^{(\alpha+\beta i)t} = 2e^{\alpha t}[a\cos\beta t + b\sin\beta t]}$$

$$Y(t) = 2e^{1.0246t}[0.0338\cos 3.339t + 0.239\sin 3.339t]$$
$$+ 0.8125\,e^{-2.0493t} + 0.12.$$

Steady-state response $= 0.12 + 2e^{1.0246t}[0.0338\cos 3.339t + 0.239\sin 3.339t]$

Transient - response $= 0.8125\,e^{-2.0493t}$.

b)

$$u(t) = t^2 \Rightarrow U(s) = 2/s^3$$

$$Y(s) = \frac{(2s+1)\frac{2}{s^3} + s^2 + 8}{s^3 + 8s + 25}$$

$$Y(s) = \frac{s^5 + 8s^3 + 4s + 2}{s^6 + 8s^4 + 25s^3}$$

Using matlab;

$$Y(s) = \frac{0.2079 - 0.1085i}{s - 1.0246 - 3.3391i} + \frac{0.2079 + 0.1085i}{s - 1.0246 + 3.3391i} + \frac{0.6272}{s + 2.0493}$$

$$- \frac{0.043}{s} + \frac{0.1344}{s^2} + \frac{0.08}{s^3}$$

$$Y(t) = (0.2079 - 0.1085i)e^{(1.0246 + 3.339i)t} + (0.2079 + 0.1085i)e^{(1.0246 - 3.339i)t}$$

$$+ 0.6272 e^{-2.0493} - 0.043 + 0.1344t + 0.04t^2$$

$$Y(t) = 2e^{1.0246t}[0.2079 \cos 3.339t + 0.1085 \sin 3.339t]$$

$$+ 0.6272 e^{-2.0493t} - 0.043 + 0.1344t + 0.04t^2$$

Steady-state response: $-0.043 + 0.1344t + 0.04t^2 + 2e^{1.0246t} \times [0.2079 \cos 3.339t + 0.1085 \sin 3.339t]$

Transient response: $0.6272 e^{-2.0493t}$

c)

$$u(t) = \cos 3t \Rightarrow u(s) = \frac{s}{s^2+9}$$

$$y(s) = \frac{2s+1 \cdot \frac{s}{s^2+9} + s^2+8}{s^3+8s+25}$$

$$Y(s) = \frac{s^4+17s^2+72+2s^2+s}{(s^2+9)(s^3+8s+25)}$$

$$Y(s) = \frac{s^4+19s^2+s+72}{s^5+17s^3+25s^2+72s+225}$$

Using Matlab:

$$Y(s) = \frac{0.1867-0.24i}{s-1.0246-3.3391i} + \frac{0.1867+0.24i}{s-1.0246+3.3391i} + \frac{0.0055+0.1207i}{s-0-3i}$$

$$+ \frac{0.0055-0.1207i}{s-0+3i} + \frac{0.6156}{s+2.0493}$$

$$Y(t) = (0.1867-0.24i)e^{(1.0246+3.3391i)t} + (0.1867+0.24i)e^{(1.0246-3.3391i)t}$$

$$+ (0.0055+0.1207i)e^{(0+3i)t} + (0.0055+0.1207i)e^{(0-3i)t}$$

$$+ 0.6156\, e^{-2.0493t}$$

$$Y(t) = 2e^{1.0246t}\left[0.1867\cos 3.3391t + 0.24\sin 3.3391t\right]$$

$$+ 2\left[0.0055\cos 3t + 0.1207\sin 3t\right] + 0.6156\, e^{-2.0493t}$$

Steady-state response: $2e^{1.0246t}\left[0.1867\cos 3.3391t + 0.24\sin 3.3391t\right] + \left[0.0110\cos 3t + 0.2414\sin 3t\right]$

Transient response: $0.6156\, e^{-2.0493t}$

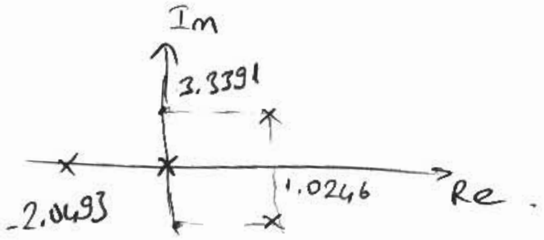
(ii)

a)

$$\frac{Y(s)}{U(s)} = \frac{2s+1}{s^3+8s+25}$$

$$Y(s) = \frac{6s+3}{s(s^3+8s+25)}$$

Pole location:

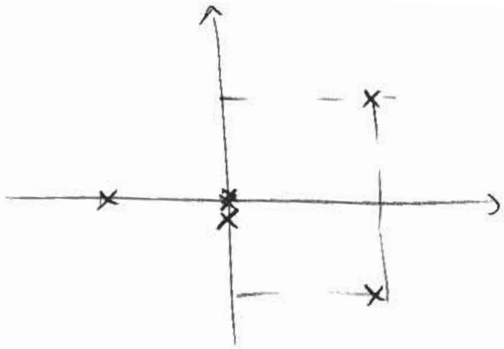


$\Rightarrow$ limit doesn't exists.

b)

$$\frac{Y(s)}{U(s)} = \frac{2s+1}{s^3+8s+25}$$

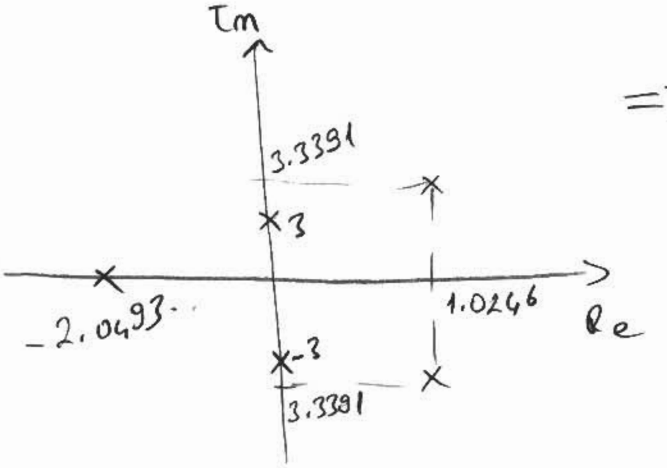
$$Y(s) = \frac{4s+2}{s^6+8s^4+25s^3}$$



$\Rightarrow$ limit doesn't exists.

c)

$$Y(s) = \frac{s^4 + 19s^2 + s + 72}{s^5 + 17s^3 + 25s^2 + 72s + 225}$$



$\Rightarrow$ limit doesn't exists.

Problem 2)

(i)

$$Y(s) = \frac{1}{s(2s^2+s+4)}$$

poles are $s_1 = 0$ $\quad s_2 = -0.25 + 1.3919i$

$s_3 = -0.25 - 1.3919i$ $\quad$ limit exists

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 1/4$$

(ii)

$$Y(s) = \frac{1}{s^3+3s-4}$$

poles are $s_1 = 1$ $s_2 = -0.5+1.9365i$

unstable pole.

$s_3 = -0.5-1.9365i$

limit doesn't exists.

(iii)

$$Y(s) = \frac{s+2}{s^2(4s^2+2)}$$

poles are $s_{1,2} = 0$ $s_{2,3} = \mp 0.7071i$

limit doesn't exists.

(iv)

$$Y(s) = \frac{4}{(s-1)s^2}$$

poles are $s_{1,2} = 0$ $s_3 = 1$

limit doesn't exists

(v)

$$Y(s) = \frac{s}{(s+2)(s^2+4)}$$

poles are $s_1 = -2$ $s_{2,3} = \mp 2i$

limit doesn't exists

Problem 3)

(i) $G(s) = \frac{9}{s+3} \Rightarrow$ pole: $p = -3$

time constant $\tau = -1/p = 1/3$

DC gain $K = G(0) = 3$

(ii) $G(s) = \frac{1}{s+0.5} \Rightarrow$ pole : $p = -0.5$

time constant $\tau = -1/p = 2$

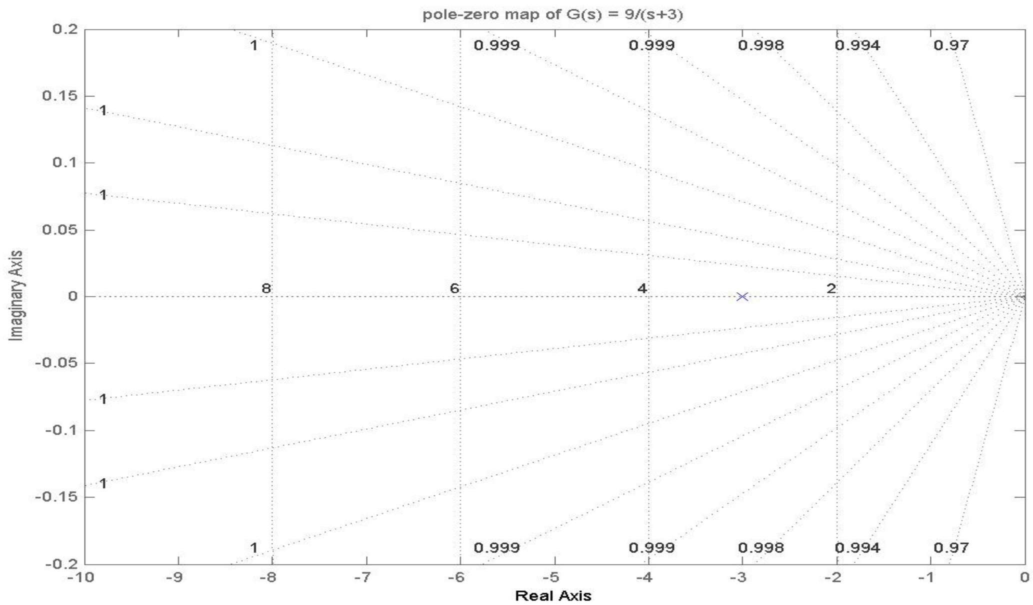
DC gain $K = G(0) = 2$.

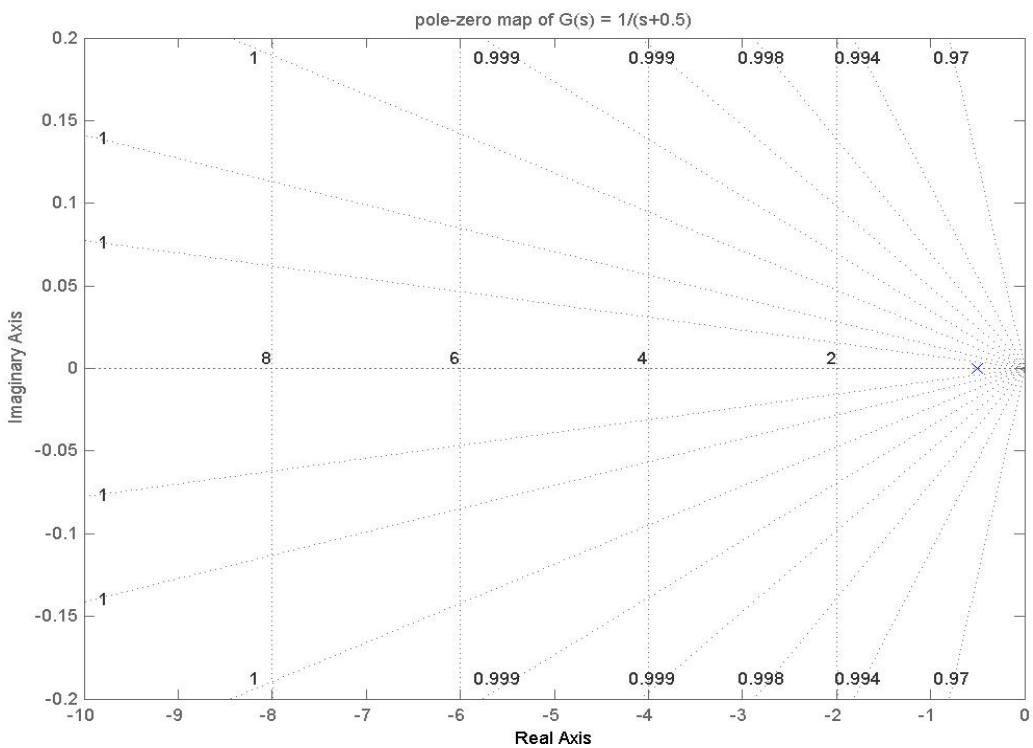
(iii) $G(s) = \frac{14}{s+1} \Rightarrow$ pole : $p = -1$

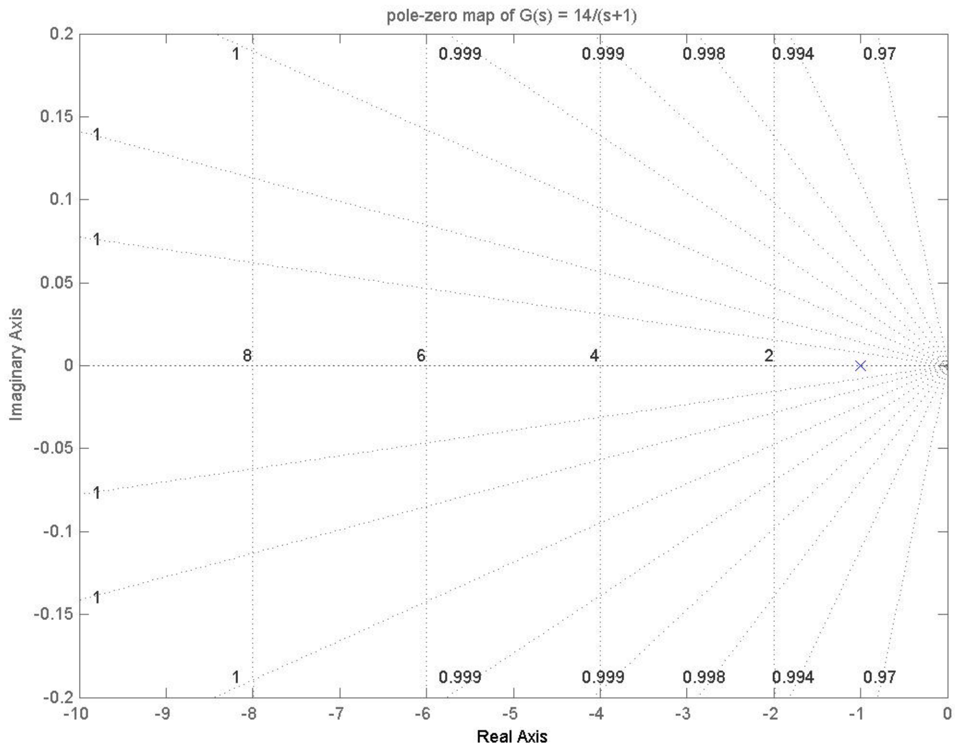
time constant $\tau = -1/p = 1$

DC gain $K = G(0) = 14$.

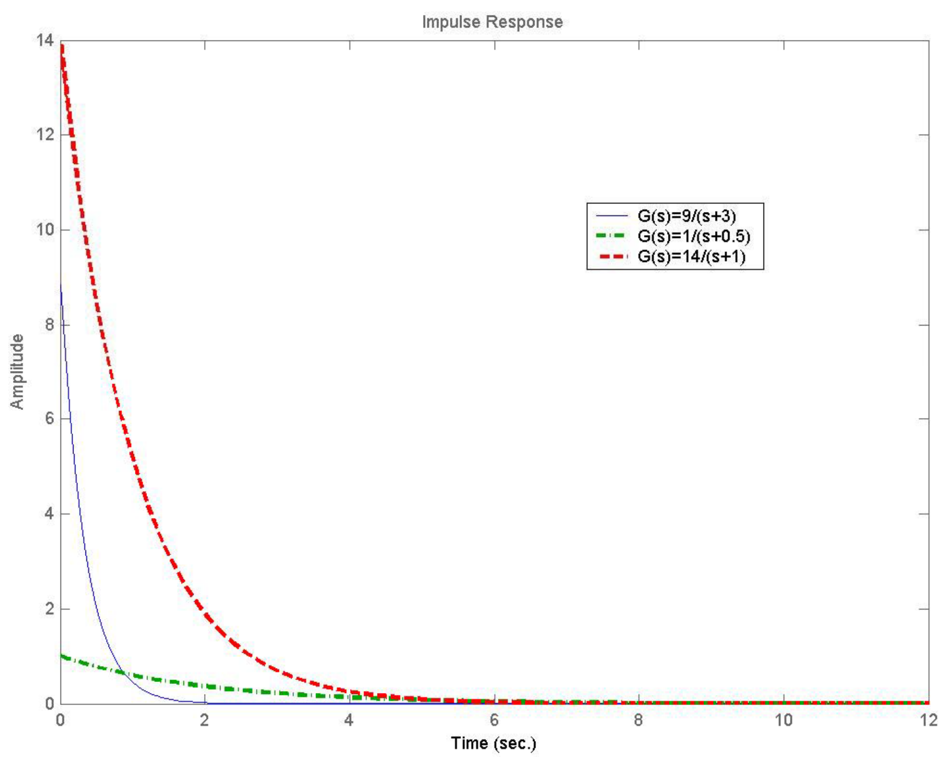
Problem # 4(i)

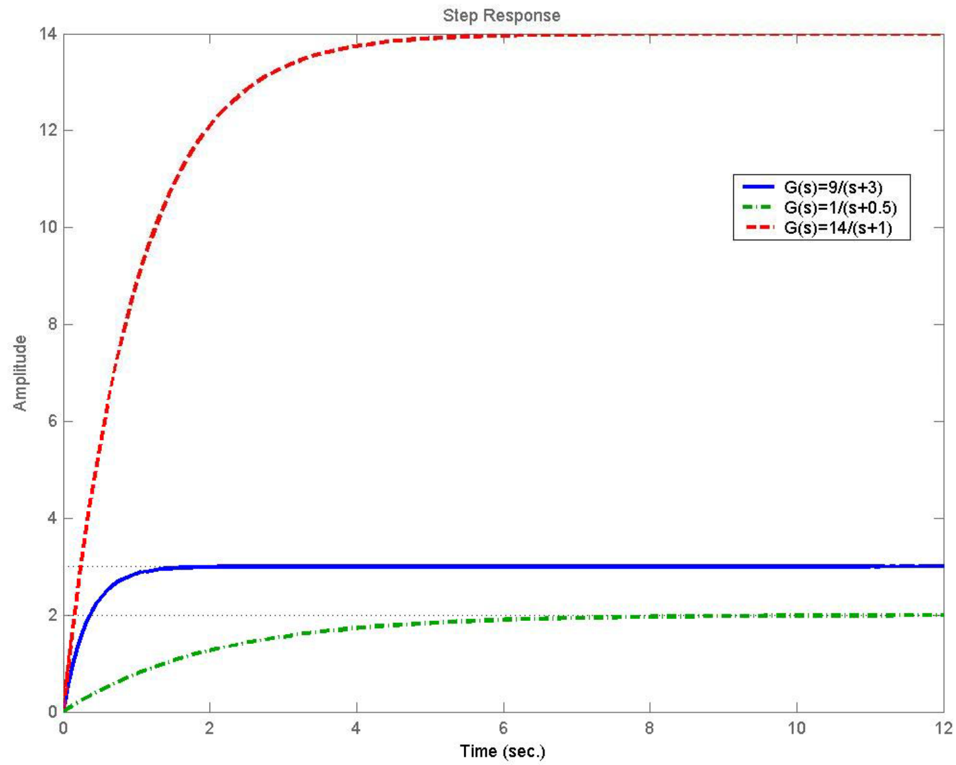

pole-zero map of G(s) = 9/(s+3)
Imaginary Axis
Real Axis
0.999
0.998
0.994
0.97
8
6
4
2


pole-zero map of G(s) = 1/(s+0.5)
Imaginary Axis
Real Axis
0.999
0.998
0.994
0.97
8
6
4
2


pole-zero map of G(s) = 14/(s+1)
Imaginary Axis
Real Axis
0.999
0.998
0.994
0.97
8
6
4
2

Problem # 4(ii)


Impulse Response
Amplitude
Time (sec.)
G(s)=9/(s+3)
G(s)=1/(s+0.5)
G(s)=14/(s+1)


Step Response
Amplitude
Time (sec.)
G(s)=9/(s+3)
G(s)=1/(s+0.5)
G(s)=14/(s+1)
0
2
4
6
8
10
12
14

iii) As τ increases, the poles get closer to the imaginary axis. So, the response of the system becomes slower, i.e. the time to asymptotically reach the steady-state (for step response) increases, and the time to asymptotically decay to zero (for impulse response) also increases.

As DC gain increases, the steady state value of step response increases. In fact, the steady state value of the response to a unit step is the DC gain.

Initial jump of the impulse response depends on the ratio $\frac{K}{\tau}$. That is, if DC gain increases, the jump becomes bigger, but if τ increases, the jump becomes smaller.

MAE 4310 Design Project Assignment

1) • Aileron Servo Motor

$$0.1\ \dot{\delta}_a(t) + \delta_a(t) = V_{servo}(t)$$

char. Eq. : $0.1\,\lambda + 1 = 0$

$\Rightarrow$ pole : $\lambda = -10$

$\Rightarrow$ The system is internally and BIBO stable. The free response is an exponential decay.

• Lateral Dynamics

char. Eq. : $\lambda^4 + 1.286\,\lambda^3 + 0.543\,\lambda^2 + 0.6114\,\lambda + 0.034 = 0$

$\Rightarrow$ poles :

$$\lambda_1 = -1.2255$$

$$\lambda_{2,3} = 0.0019 \pm j\,0.6903$$

$$\lambda_4 = -0.0582$$

$\Rightarrow$ The system is internally and BIBO unstable. The free response is an oscillatory exponential growth.

TRANSFER FUNCTIONS

- from V_{servo} to δ_a , $T_1(s) = T_{V_{servo} \to \delta_a}(s)$

$$0.1\,\dot{\delta}_a(t) + \delta_a(t) = V_{servo}(t)$$

$\mathcal{L}$

$$(0.1s+1)\,\delta_a(s) = V_{servo}(s)$$

$$\Rightarrow \boxed{T_1(s) = \frac{\delta_a(s)}{V_{servo}(s)} = \frac{1}{0.1s+1}}$$

- from δ_a to p , $T_2(s) = T_{\delta_a \to p}(s)$

Taking Laplace transform of the equation of the Lateral Dynami

$$(s^4 + 1.286\,s^3 + 0.543\,s^2 + 0.6114\,s + 0.034)\;p(s) =$$

$$= (0.23\,s^3 + 0.079\,s^2 + 0.0789\,s + 0.0001)\;\delta_a(s)$$

$\Rightarrow$

$$\boxed{T_2(s) = \frac{p(s)}{\delta_a(s)} = \frac{0.23\,s^3 + 0.079\,s^2 + 0.0789\,s + 0.0001}{s^4 + 1.286\,s^3 + 0.543\,s^2 + 0.6114\,s + 0.034}}$$

- from p to ϕ, $T_3(s) = T_{p \to \phi}(s)$

$$\dot{\phi}(t) = p(t)$$

$$\mathcal{L} \quad s\phi(s) = p(s)$$

$$\Rightarrow \quad \boxed{T_3(s) = \frac{\phi(s)}{p(s)} = \frac{1}{s}}$$

- from ϕ to ψ, $T_4(s) = T_{\phi \to \psi}(s)$

$$\dot{\psi}(t) = \frac{g}{U_0}\phi(t)$$

$$\mathcal{L} \quad s\psi(s) = \frac{g}{U_0}\phi(s)$$

$$\Rightarrow \quad \boxed{T_4(s) = \frac{\psi(s)}{U(s)} = \frac{g}{U_0} \cdot \frac{1}{s}}$$

- from p to V_{gyro}, $T_5(s) = T_{p \to V_{gyro}}(s)$

$$V_{gyro}(t) = 0.5\, p(t)$$

$$\mathcal{L} \quad V_{gyro}(s) = 0.5\, p(s)$$

$$\Rightarrow \quad \boxed{T_5(s) = \frac{V_{gyro}(s)}{p(s)} = 0.5}$$

Block Diagram :