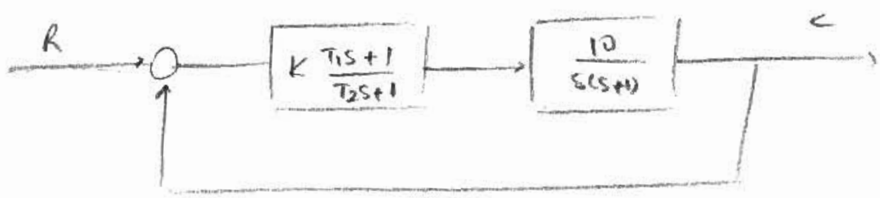
MAE 4310 SOLUTION TO HW # 7

B.7.7)



$$\frac{C(s)}{R(s)} = \frac{K \cdot \frac{T_1 s+1}{T_2 s+1} \cdot \frac{10}{s(s+1)}}{1 + k\,\frac{T_1 s+1}{T_2 s+1} \cdot \frac{10}{s(s+1)}}$$

$$= \frac{10k\,(T_1 s+1)}{(s^2+s)(T_2 s+1) + 10kT_1 s + 10K}$$

Char. poly. $:= \; T_2 s^3 + (1+T_2)s^2 + (10kT_1+1)s + 10K$

Desired dominant closed loop poles: $\zeta = 0.5$, $\omega_n = 3$ rad/sec

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 3s + 9.$$

$$T_2 s^3 + (1+T_2)s^2 + (10kT_1+1)s + 10k = (s^2+3s+9)(as+b)$$

To get a stable closed loop

$$s = -\frac{b}{a} < 0 \;=\; \frac{b}{a} > 0$$

$$(s^2+3s+9)(as+b) = as^3 + (3a+b)s^2 + (3b+9a)s + 9b.$$

$$\left.\begin{aligned} T_2 &= a \\ T_2+1 &= 3a+b \end{aligned}\right\} \qquad \frac{3a+b}{a} = \frac{T_2+1}{T_2} \qquad \frac{b}{a}+3 = 1+\frac{1}{T_2}$$

$$\boxed{\frac{b}{a} = \frac{1}{T_2} - 2}$$

$$10kT_1 + 1 = 3b + 9a$$

$$10k = 9b.$$

from the condition $\frac{b}{a} > 0$

$$\frac{1}{T_2} - 2 > 0 \qquad \boxed{T_2 < 1/2.}$$

for $T_2 = 1/3$

$a = 1/3$, $b = 1/3$ $\Rightarrow$ other pole is located at $\boxed{s=-1}$

$\boxed{k = 0,3.}$, $\boxed{T_1 = 1.}$

B-7-8)

closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K(Ts+1)}{s(s+2)+K(Ts+1)}$$

Since the closed-loop poles are located at $s=-2\pm j2$

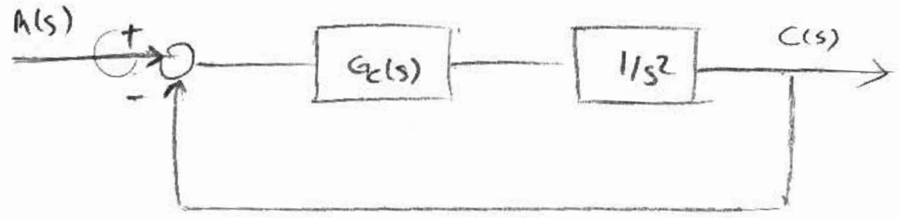
$$s(s+2)+K(Ts+1) = (s+2+j2)(s+2-j2)$$

$$s^2+(2+KT)s+K = s^2+4s+8$$

$$2+KT=4 \quad , \quad \boxed{K=8}$$

$$\boxed{T=0.25}$$

B_7_10)



Dominant desired-closed loop poles $s = -1 \mp 1j$

$$(s+1+j)(s+1-j) = s^2 + 2s + 2$$

$$G_c(s) = k\,\frac{s+a}{s+b}$$

$$\frac{C(s)}{R(s)} = \frac{k\,\frac{s+a}{s+b}\,{}^1/_{s^2}}{1 + k\,\frac{s+a}{s+b}\cdot\frac{1}{s^2}}$$

$$= \frac{k(s+a)}{s^3 + bs^2 + ks + ka}$$

char. poly $= s^3 + bs^2 + ks + ka$.

$$s^3 + bs^2 + ks + ka = (s^2 + 2s + 2)(\alpha s + \beta)$$

Similar to B_7_7 $\quad \frac{\beta}{\alpha} > 0$.

C

$$s^3 + bs^2 + ks + ka = \alpha s^3 + (2\alpha + \beta)s^2 + (2\alpha + 2\beta)s + 2\beta$$

$$\left.\begin{array}{l} 1 = \alpha \\ b = 2\alpha + \beta \end{array}\right\} \quad \frac{2\alpha+\beta}{\alpha} = b \Rightarrow \frac{\beta}{\alpha} = b - 2$$

$k = 2(\alpha + \beta)$

$ka = 2\beta$

from the condition $\frac{\beta}{\alpha} > 0$

$b - 2 > 0 \qquad b > 2$

pole @ $s = -\frac{\beta}{\alpha}$ should be located far enough from the dominant poles.

for $b = 5$

$\alpha = 1$, $\beta = 3$ $\quad \boxed{s = -3}$

$\boxed{k = 8} \qquad a = 6/8 \Rightarrow \boxed{a = 0.75}$

Thus,

$$\boxed{G_c(s) = 8\,\frac{s + 0.75}{s + 5}}$$

MATLAB CODE

```
num=[0 0 1];
den=[1 0 1];
numc=[8 6];
denc=[1 5 8 6];
t=0:0.02:10;
c1=step(num,den,t);
c2=step(numc,denc,t);
plot(t,c1,'.',t,c2,'-')
grid
title('Unit step responses of uncompensated and compensated systems')
xlabel('t(sec)')
ylabel('Outputs')
text(1.9,0.85,'Compensated system')
text(4.1,1.65,'Uncompensated system')
```