MAE3360 HW#5 (key assignment) Solutions

DATE: 07/20/2009

SET 3.8

Problem 2

Solutions: From $20x'' + kx = 0$ we obtain

$$x = c_1 \cos\frac{1}{2}\sqrt{\frac{k}{5}}\,t + c_2 \sin\frac{1}{2}\sqrt{\frac{k}{5}}\,t$$

so that the frequency $2/\pi = \frac{1}{4}\sqrt{\frac{k}{5}}\Big/\pi$

and $k = 320\,N/m$. If $80x'' + 320x = 0$

then $x = c_1\cos 2t + c_2 \sin 2t$

so the frequency is $2/2\pi = 1/\pi$ cycles/s

Problem 4

Solutions: From $\frac{3}{4}x'' + 72x = 0$, $x(0) = 0$, $x'(0) = 2$

$x(t) = c_1 \cos 4\sqrt{6}t + c_2 \sin 4\sqrt{6}t$,

applying initial condition, we get

$$x = \frac{\sqrt{6}}{12}\sin 4\sqrt{6}t$$

Problem 8

Solutions: From $x'' + 16x = 0$, $x(0) = -1$, $x'(0) = -2$

we obtain

$$x = -\cos 4t - \frac{1}{2}\sin 4t$$
$$= \frac{\sqrt{5}}{2}\cos(4t - 3.605)$$

so the period is $T = 2\pi / 4 = \pi / 2$

the amplitude is $(\sqrt{5}/2)$ feet.

In 4π seconds, $4\pi / T = 8$

it will make 8 complete cycles

Problem 11

Solutions: From $2x'' + 200x = 0$, $x(0) = -\frac{2}{3}$, $x'(0) = 5$

(a) $x = -\frac{2}{3}\cos 10t + \frac{1}{2}\sin 10t = \frac{5}{6}\sin(10t - 0.927)$

(b) The amplitude is $\frac{5}{6}$ ft

the period is $2\pi / 10 = \pi / 5$

(c) $3\pi = \pi k / 5$, $k = 15$ cycles

(d) If $x = 0$ and the weight is moving downward for the second time, then $10t - 0.927 = 2\pi$

$t = 0.721\,s$

(e) $\frac{5}{6}\sin(10t - 0.927) = \pm\frac{5}{6}$

$10t - 0.927 = \frac{\pi}{2} + n\pi$

$t = (2n+1)\pi/20 + 0.0927, \quad n = 0, 1, 2 \cdots$

(f) $x(3) = -0.597\,ft$

(g) $x'(3) = -5.814\,ft/s$

(h) $x''(3) = 59.702\,ft/s^2$

(i) If $x = 0$ then $t = \frac{1}{10}(0.927 + n\pi), \; n = 0, 1, 2 \cdots$

The velocity at these times is $x' = \pm 8.33\,ft/s$

(j) If $x = 5/12$ then $t = \frac{1}{10}(\pi/6 + 0.927 + 2n\pi)$

and $t = \frac{1}{10}(5\pi/6 + 0.927 + 2n\pi), \; n = 0, 1, 2 \cdots$

(k) If $x = 5/12$ and $x' < 0$, then

$t = \frac{1}{10}(5\pi/6 + 0.927 + 2n\pi), \; n = 0, 1, 2 \cdots$

Problem 23

Soluteions:

(a) From $x'' + 10x' + 16x = 0$, $x(0) = 1$, $x'(0) = 0$

we obtain $x = \frac{4}{3} e^{-2t} - \frac{1}{3} e^{-8t}$

(b) From $x'' + 10x' + 16x = 0$, $x(0) = 1$, $x'(0) = -12$

then $x = -\frac{2}{3} e^{-2t} + \frac{5}{3} e^{-8t}$

Phroblem 25

Soluteions:

(a) From $0.1x'' + 0.4x' + 2x = 0$, $x(0) = -1$, $x'(0) = 0$

we obtain $x = e^{-2t}\left[-\cos 4t - \frac{1}{2}\sin 4t\right]$

(b) $x = \frac{\sqrt{5}}{2} e^{-2t} \sin(4t + 4.25)$

(c) If $x = 0$ then $4t + 4.25 = 2\pi, 3\pi, 4\pi \cdots$

so that the first time heading upwarde

is $t = 1.294$ seconds

Problem 29

Solutions:

If $\frac{1}{2}x'' + \frac{1}{2}x' + 6x = 10\cos 3t$

$x(0) = 2, \quad x'(0) = 0$ then

$$x_c = e^{-t/2}\left(c_1 \cos\frac{\sqrt{47}}{2}t + c_2 \sin\frac{\sqrt{47}}{2}t\right)$$

$$x_p = \frac{10}{3}(\cos 3t + \sin 3t)$$

applying the initial condition, we obtain

$$x = e^{-t/2}\left(-\frac{4}{3}\cos\frac{\sqrt{47}}{2}t - \frac{64}{3\sqrt{47}}\sin\frac{\sqrt{47}}{2}t\right)$$

$$+ \frac{10}{3}(\cos 3t + \sin 3t)$$