MAE3360 HW#6 Solutions DATE: 07/29/09

SET 4.2

Problem 23

Solutions:

$$\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{1}{s-2} - \frac{1}{s-3} + \frac{1}{2}\cdot\frac{1}{s-6}\right\}$$

$$= \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$$

Problem 25

Solutions:

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s^2+5)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{5}\cdot\frac{1}{s} - \frac{1}{5}\cdot\frac{s}{s^2+5}\right\}$$

$$= \frac{1}{5} - \frac{1}{5}\cos\sqrt{5}t$$

Problem 34

Solutions: The Laplace transform of the initial-value problem is

$$s\mathcal{L}\{y\} - \mathcal{L}\{y\} = \frac{2s}{s^2+25}$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{2s}{(s-1)(s^2+25)} = \frac{1}{13}\cdot\frac{1}{s-1} - \frac{1}{13}\cdot\frac{s}{s^2+25} + \frac{5}{13}\cdot\frac{5}{s^2+25}$$

Thus $y = \frac{1}{13}e^t - \frac{1}{13}\cos 5t + \frac{5}{13}\sin 5t$

Problem 38

Solutions: The Laplace transform of the initial-value problem is

$$s^2\mathcal{L}\{y\} + 9\mathcal{L}\{y\} = \frac{1}{s-1}$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{1}{(s-1)(s^2+9)} = \frac{1}{10}\cdot\frac{1}{s-1} - \frac{1}{10}\cdot\frac{1}{s^2+9} - \frac{1}{10}\cdot\frac{s}{s^2+9}$$

Thus $y = \frac{1}{10}e^t - \frac{1}{30}\sin 3t - \frac{1}{10}\cos 3t$

Problem 40

Solutions: The Laplace transform of the initial-value problem is

$$s^3\mathcal{L}\{y\} - s^2(0) - sy'(0) - y''(0) + 2[s^2\mathcal{L}\{y\} - sy(0) - y'(0)]$$
$$- [s\mathcal{L}\{y\} - y(0)] - 2\mathcal{L}\{y\} = \frac{3}{s^2+9}$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{s^2+12}{(s-1)(s+1)(s+2)(s^2+9)}$$

$$= \frac{13}{60}\cdot\frac{1}{s-1} - \frac{13}{20}\cdot\frac{1}{s+1} + \frac{16}{39}\cdot\frac{1}{s+2} + \frac{3}{130}\cdot\frac{s}{s^2+9} - \frac{1}{65}\cdot\frac{3}{s^2+9}$$

Thus

$$y = \frac{13}{60}e^{t} - \frac{13}{20}e^{-t} + \frac{16}{39}e^{-2t} + \frac{3}{130}\cos 3t - \frac{1}{65}\sin 3t$$

Set 4.3

Problem 13

Solutions: $\mathcal{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+1^2}\right\} = e^{3t}\sin t$

Problem 17

Solutions: $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1-1}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{1}{(s+1)^2}\right\}$

$$= e^{-t} - te^{-t}$$

Problem 23

Solutions: The Laplace transform of the differential equation is

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 2[s\mathcal{L}(y) - y(0)] + \mathcal{L}\{y\} = 0$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{s+3}{(s+1)^2} = \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

Thus $$y = e^{-t} + 2te^{-t}$$

Problem 28

Solutions: The Laplace transform of the differential equation is

$$2[s^2\mathcal{L}\{y\} - sy(0)] + 20[s\mathcal{L}\{y\} - y(0)] + 51\mathcal{L}\{y\} = 0$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{4s+40}{2s^2+20s+51} = \frac{2s+20}{s^2+10s+51/2}$$

$$= \frac{2s+20}{(s+5)^2+1/2} = \frac{2(s+5)}{(s+5)^2+1/2} + \frac{10}{(s+5)^2+1/2}$$

Thus

$$y = 2e^{-5t}\cos(t/\sqrt{2}) + 10\sqrt{2}e^{-5t}\sin(t/\sqrt{2})$$