MAE 3360 HW #1
ENGINEERING ANALYSIS

Exercises 2.3 — LINEAR EQUATIONS

3. $\frac{dy}{dx} + y = e^{3x}$

$P(x) = 1$
$f(x) = e^{3x}$

Integrating factor is $e^{\int dx} = e^{x}$

$$e^{x}\frac{dy}{dx} + e^{x}y = e^{4x}$$

$$\frac{d}{dx}\left(e^{x}y\right) = e^{4x}$$

Integrating both sides

$$\int \frac{d}{dx}\left(e^{x}y\right)dx = \int e^{4x}dx$$

$$\therefore \quad e^{x}y = \frac{1}{4}e^{4x} + c$$

$$\therefore \boxed{y = \frac{1}{4}e^{3x} + ce^{-x}} \quad \text{for } -\infty < x < \infty$$

ce^{-x} is the transient term.

5. $\frac{dy}{dx} + 3x^2y = x^2$

$P(x) = 3x^2$

$f(x) = x^2$

Integrating factor is $e^{\int 3x^2 dx} = e^{x^3}$

$e^{x^3}\frac{dy}{dx} + e^{x^3}\cdot 3x^2y = e^{x^3}\cdot x^2$

$\frac{d}{dx}(e^{x^3}y) = e^{x^3}\cdot x^2$

Integrating both sides

$\int \frac{d}{dx}(e^{x^3}y)\,dx = \int e^{x^3}\cdot x^2\,dx$

$\therefore \boxed{y = \frac{1}{3} + ce^{-x^3}}$ for $-\infty < x < \infty$

ce^{-x^3} is the transient term.

10. $x\frac{dy}{dx} + \frac{2}{\cancel{x}}y = 3$

$\therefore \frac{dy}{dx} + \frac{2}{x}y = \frac{3}{x}$

$P(x) = \frac{2}{x}$

$f(x) = \frac{3}{x}$

Integrating factor is $e^{\int \frac{2}{x} dx} = x^2$

$x^2 y' + 2xy = 3x$

$\frac{d}{dx}[x^2 y] = 3x$

$\int \frac{d}{dx}(x^2 y)\, dx = \int 3x\, dx$

$x^2 y = \frac{3}{2}x^2 + C$

$\therefore \boxed{y = \frac{3}{2} + Cx^{-2}}$ for $0 < x < \infty$

25. $\frac{dy}{dx} + \frac{1}{x} y = \frac{1}{x} e^x$

$P(x) = \frac{1}{x}$

$f(x) = \frac{1}{x} e^x$

Integrating factor is $e^{\int \frac{1}{x} dx} = x$

$x\frac{dy}{dx} + y = e^x$

$\frac{d}{dx}(xy) = e^x$

$$\int \frac{d}{dx}(xy)\,dx = \int e^x dx$$

$$xy = e^x + c$$

$$\therefore \boxed{y = \frac{1}{x}e^x + \frac{c}{x}} \quad \text{for } 0 < x < \infty$$

$$y(1) = 2$$

$$\therefore 2 = \frac{e^1}{1} + \frac{c}{1} = e + c$$

$$\therefore c = 2 - e$$

$$\therefore \boxed{y = \frac{1}{x}e^x + \frac{2-e}{x}}$$

Exercises 2.4 — EXACT EQUATIONS.

1. $(2x-1)dx + (3y+7)dy = 0$

$M = 2x-1$ $\qquad\qquad N = 3y+7$

$\frac{\partial M}{\partial y} = 0$ $\qquad\qquad \frac{\partial N}{\partial x} = 0$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Hence, it is exact.

$$\frac{\partial g}{\partial x} = 2x-1$$

$$\therefore \int \frac{\partial g}{\partial x}\,dx = g(x,y) = x^2 - x + h(y)$$

$$\frac{\partial g}{\partial y} = h'(y) = N$$

$$\therefore h'(y) = 3y+7$$

$$\therefore h(y) = \frac{3}{2}y^2 + 7y$$

Hence, the solution is

$$\boxed{x^2 - x + \frac{3}{2}y^2 + 7y = C}$$

3. $(5x+4y)dx + (4x-8y^3)dy = 0$

$M = 5x+4y$ $\qquad$ $N = 4x - 8y^3$

$\frac{\partial M}{\partial y} = 4$ $\qquad$ $\frac{\partial N}{\partial x} = 4$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Hence, it is exact.

$$\frac{\partial g}{\partial x} = 5x+4y$$

$$g(x,y) = \int \frac{\partial g}{\partial x} \cdot dx = \int (5x+4y)\, dx$$

$$\therefore g(x,y) = \frac{5}{2}x^2 + 4xy + h(y)$$

$$\frac{\partial g}{\partial y} = \cancel{5x} + 4\overset{x}{\cancel{y}} + h'(y) = N$$

$$\cancel{5x+4y} + \quad 4x + h'(y) = 4x - 8y^3$$

$$\therefore h'(y) = -8y^3$$

$$\therefore h(y) = -2y^4$$

Hence, the solution is

$$\boxed{\frac{5}{2}x^2 + 4xy - 2y^4 = c}$$

9. $(x - y^3 + y^2 \sin x)\, dx = (3xy^2 + 2y\cos x)\, dy$

$(x - y^3 + y^2 \sin x)\, dx + (-3xy^2 - 2y\cos x)\, dy = 0$

$M = x - y^3 + y^2 \sin x \qquad N = -3xy^2 - 2y\cos x$

$$\frac{\partial M}{\partial y} = -3y^2 + 2y\sin x \qquad \frac{\partial N}{\partial x} = -3y^2 + 2y\sin x$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Hence, it is exact

$$\frac{\partial g}{\partial x} = x - y^3 + y^2 \sin x$$

$$\therefore g(x,y) = \int \frac{\partial g}{\partial x}\, dx = \frac{x^2}{2} - y^3 x - y^2 \cos x + h(y)$$

$$\frac{\partial g}{\partial y} = -3y^2 x - 2y\cos x + h'(y) = N$$

$$\therefore -3y^2 x - 2y\cos x + h'(y) = -3xy^2 - 2y\cos x$$

$$h'(y) = 0$$

$$\therefore h(y) = 0$$

Hence, the solution is

$$\boxed{xy^3 + y^2\cos x - \frac{1}{2}x^2 = c}$$

21. $(x+y)^2 dx + (2xy + x^2 - 1) dy = 0$

$M = (x+y)^2 = x^2 + 2xy + y^2 \qquad N = 2xy + x^2 - 1$

$\frac{\partial M}{\partial y} = 2(x+y) \qquad \frac{\partial N}{\partial x} = 2(x+y)$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Hence, it is exact.

$\frac{\partial g}{\partial x} = x^2 + 2xy + y^2$

$\int \frac{\partial g}{\partial x} \cdot dx = g(x,y) = \frac{1}{3}x^3 + x^2 y + xy^2 + h(y)$

$g(x,y) = \frac{1}{3}x^3 + x^2 y + xy^2 + h(y)$

$\frac{\partial g}{\partial y} = x^2 + 2xy + h'(y) = N$

$x^2 + 2xy + h'(y) = 2xy + x^2 - 1$

$\therefore h'(y) = -1$

$\therefore h(y) = -y$

Hence, the solution is

$$\boxed{\frac{1}{3}x^3 + x^2y + xy^2 - y = C}$$

If $y(1) = 1$

$$\frac{1}{3} \times 1 + 1 \times 1 + 1 \times 1^2 - 1 = C$$

$$C = \frac{4}{3}$$

Hence, solution of the initial value problem is

$$\boxed{\frac{1}{3}x^3 + x^2y + xy^2 - y = \frac{4}{3}}$$