SUB : MAE 3360

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SOLUTIONS TO ASSIGNMENT # 01

DUE ON : [redacted]01/30/08

QUESTIONS :

EXERCISE : 2.3

Find the general sol. of the given d.E. Give the largest interval over which the general sol. is defined. Determine whether there ~~are~~ ~~any~~ transient terms in the general solution.

(7) $x^2 y' + xy = 1$

(11) $x \dfrac{dy}{dx} + 4y = x^3 - x$

(13) $x^2 y' + (x(x+2))y = e^x$ ~~(25)~~

(25) Solve the given initial value problem. Give the largest interval "I" over which the sol. is defined.

$$xy' + y = e^x \; ; \; y(1) = 2$$

EXERCISE : 2.4

Determine whether the given d.E is exact. If it is exact, Solve it.

(5) $(2xy^2 - 3)\,dx + (2x^2 y + 4)\,dy = 0$

(13) $x \dfrac{dy}{dx} = 2xe^x - y + 6x^2$

(19) $(4t^3 y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$

(23) Solve the given Initial Value Problem:

$$(4y + 2t - 5)\,dt + (6y + 4t - 1)\,dy = 0 \quad ; \; y(-1) = 2$$

## SOLUTIONS

### EXERCISE : 2.3

Find the general Sol. of the given differential equation. Give the largest interval over which the general solution is defined. Determine whether there are any transient terms in the general solution.

(7) $x^2y' + xy = 1$

Sol:- Given $x^2y' + xy = 1$

$$\Rightarrow y' + \frac{y}{x} = \frac{1}{x^2} \quad \text{(Dividing both sides by } x^2\text{)}$$

On Comparision with $y' + P(x)\,y = f(x)$,

we have $P(x) = \frac{1}{x}$ ; $f(x) = \frac{1}{x^2}$

$\therefore$ Integrating factor: $e^{\int P(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$$\therefore \frac{d}{dx}[xy] = \frac{1}{x^2}(x) \Rightarrow \frac{d}{dx}(xy) = \frac{1}{x}$$

on integrating,

$$xy = \ln x + c$$

$$\Rightarrow y = \frac{1}{x}\ln x + \frac{c}{x} \quad \text{for } 0 < x < \infty$$

(11) $xy' + 4y = x^3 - x$

Sol:- Given $xy' + 4y = x^3 - x$

$$\Rightarrow y' + \left(\frac{4}{x}\right)y = x^2 - 1 \quad \text{—(1)}$$

(Dividing both the sides by $x$)

Comparing eq. ① with $y' + P(x)y = f(x)$, we have

$$P(x) = \frac{4}{x} \; ; \; f(x) = x^2 - 1$$

$\therefore$ I.f : $e^{\int P(x)\,dx} = e^{\int \frac{4}{x}\,dx} = e^{4\ln x} = e^{\ln x^4} = x^4$

$\therefore$ The sol. is $\frac{d}{dx}\left[x^4 y\right] = x^4(x^2-1)$

on integration,

$$x^4 y = \int (x^6 - x^4)\,dx + c$$

$$\Rightarrow x^4 y = \frac{x^7}{7} - \frac{x^5}{5} + c$$

$$\Rightarrow y = \frac{x^3}{7} - \frac{x}{5} + \frac{c}{x^4} \quad \text{for } 0 < x < \infty$$

Ex: 2.3

(13) $x^2 y' + x(x+2)y = e^x$

Sol:- Given $x^2 y' + x(x+2)y = e^x$

$\Rightarrow y' + \left(1 + \frac{2}{x}\right)y = \frac{e^x}{x^2}$ — ① [Dividing both sides by $x^2$]

on Comparision with $y' + P(x)y = f(x)$,

we have $P(x) = 1 + \frac{2}{x}$ ; $f(x) = \frac{e^x}{x^2}$

$\therefore$ I.f : $e^{\int P(x)\,dx} = e^{\int (1 + \frac{2}{x})\,dx} = e^{[x + \ln x^2]} = e^x \cdot e^{\ln x^2}$

$= x^2 e^x$

$\therefore$ The Sol. is $\frac{d}{dx}\left[x^2 e^x y\right] = (x^2 e^x)\frac{e^x}{x^2}$

on integrating, $x^2 e^x y = \int e^{2x}\,dx + c$

$$\Rightarrow x^2 e^x y = \frac{e^{2x}}{2} + c$$

$$\Rightarrow y = \frac{1}{2}\left[\frac{e^{2x}}{x^2 e^x}\right] + \frac{c}{x^2 e^x}$$

$$\Rightarrow y = \frac{e^x}{2x^2} + \frac{c e^{-x}}{x^2} \quad \text{for } 0 < x < \infty.$$

The transient term is $\frac{c e^{-x}}{x^2}$.

Ex: 2.3

Solve the given initial-value problem. Give the largest interval "I" over which the solution is defined.

(25) $xy' + y = e^x$; $y(1) = 2$

Sol: Given $xy' + y = e^x$

$$\Rightarrow y' + \frac{y}{x} = \frac{e^x}{x} \quad \text{(Dividing both sides by } x)$$

on Comparision with standard form,

$$P(x) = \frac{1}{x}; \; f(x) = \frac{e^x}{x}$$

$$\therefore \text{I.f}: e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$\therefore$ The Sol. is:

$$\frac{d}{dx}[xy] = \cancel{x}\frac{e^x}{\cancel{x}}$$

On integrating,

$$xy = e^x + c$$

$$\Rightarrow y = \frac{e^x}{x} + \frac{c}{x} \quad ; \quad 0 < x < \infty$$

Given $y(1) = 2$

$$\Rightarrow 2 = e + c \Rightarrow c = e - 2$$

$$\therefore y = \frac{e^x}{x} + \frac{2-e}{x}$$

EXERCISE : 2.4

Determine whether the given differential equation is exact. If it is exact, Solve it.

(5) $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$

Sol:- Let $M = 2xy^2 - 3$ ; $N = 2x^2y + 4$

$$\Rightarrow \frac{\partial M}{\partial y} = 4xy \; ; \; \frac{\partial N}{\partial x} = 4xy$$

$$\Rightarrow M_y = N_x$$

Hence exact.

$\therefore$ There exists a function $f(x,y)$ such that

$$f_x = M \; ; \; f_y = N$$

$$f_x \equiv \frac{\partial f}{\partial x} = 2xy^2 - 3$$

on integrating,

$$f = \int (2xy^2 - 3)\,dx + h(y) = x^2y^2 - 3x + h(y)$$

differentiating both sides w.r.t 'y'

$$\Rightarrow f_y \equiv \frac{\partial f}{\partial y} = 2x^2y + h'(y)$$

but $f_y = N \Rightarrow h'(y) = 4$

on integration, $h(y) = 4y$

Ex: 2.4

(13) $x \frac{dy}{dx} = 2xe^{x} - y + 6x^{2}$

Sol:- $x \frac{dy}{dx} = 2xe^{x} - y + 6x^{2}$

$\Rightarrow x\,dy = (2xe^{x} - y + 6x^{2})\,dx$

$\Rightarrow (y - 6x^{2} - 2xe^{x})\,dx + x\,dy = 0$

Let $M = y - 6x^{2} - 2xe^{x}$ ; $N = x$

$\Rightarrow M_y = 1$ ; $N_x = 1$

$\therefore M_y = N_x \Rightarrow$ Exact.

$\therefore$ there exists a function $f(x,y)$ such that

$f_x = M$ ; $f_y = N$

$f_x = \frac{\partial f}{\partial x} = y - 6x^{2} - 2xe^{x}$

On Integration, we get

$f = xy - 2x^{3} - 2xe^{x} + 2e^{x} + h(y)$

Differentiating "f" w.r.t 'y'

$f_y = x + h'(y)$

but $f_y = N = x$

$\therefore h'(y) = 0 \Rightarrow h(y) = 0$ (or) any Constant 'c'.

$\therefore$ The Sol. is : $xy - 2x^{3} - 2xe^{x} + 2e^{x} = c$

Ex: 2.4

(19) $(4t^3y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$

Sol: Let $M = 4t^3y - 15t^2 - y$ ; $N = t^4 + 3y^2 - t$

$\Rightarrow M_y = 4t^3 - 1$ ; $N_t = 4t^3 - 1$

$\therefore M_y = N_t \Rightarrow$ Exact.

$\therefore$ There exists a function $f(t, y)$ such that

$f_t = M$ ; $f_y = N$

$$f_t = \frac{\partial f}{\partial t} = 4t^3y - 15t^2 - y$$

on Integration,

$$f = yt^4 - 5t^3 - yt + h(y)$$

Differentiating '$f$' w.r.t '$y$',

$$f_y = t^4 - t + h'(y)$$

but $f_y = N = t^4 + 3y^2 - t$

$\therefore$ on comparision,

$$h'(y) = 3y^2$$

on Integrating,

$$h(y) = y^3.$$

$\therefore$ The Sol. is

$$f \equiv yt^4 - 5t^3 - ty + y^3 = c$$

Ex: 2.3

Solve the given Initial Value problem.

(23) $(4y+2t-5)\,dt + (6y+4t-1)\,dy = 0$ ; $y(-1) = 2$

Sol: Let $M = 4y+2t-5$ ; $N = 6y+4t-1$

$\Rightarrow M_y = 4$ ; $N_t = 6$

$\therefore M_y \neq N_t \Rightarrow$ Exact.

$\therefore$ There exists a function $f(t, y)$ such that

$f_t = M$ ; $f_y = N$

$f_t = \frac{\partial f}{\partial t} = 4y + 2t - 5$

on Integration,

$f = 4yt + t^2 - 5t + h(y)$

Differentiating "$f$" w.r.t "$y$"

$\Rightarrow f_y = 4t + h'(y)$

but $f_y = N = 6y + 4t - 1$

$\Rightarrow h'(y) = 6y - 1$

on Integration,

$h(y) = 3y^2 - y$

$\therefore$ The sol is: $f \equiv 4yt + t^2 - 5t + 3y^2 - y = c$

Given $y(-1) = 2 \Rightarrow$ at $t = -1$ ; $y = 2$

$\Rightarrow c = 8$

$\therefore f \equiv 4yt + t^2 - 5t + 3y^2 - y = 8$