MAE 3360 HW # 3

ENGINEERING ANALYSIS

Exercises 3.3 – HOMOGENEOUS LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS.

3. $y'' - y' - 6y = 0$

The auxiliary equation is

$m^2 - m - 6 = 0$

$\therefore m = 3$ and $m = -2$

$\therefore \boxed{y = c_1 e^{3x} + c_2 e^{-2x}}$

5. $y'' + 8y' + 16y = 0$

The auxiliary equation is

$m^2 + 8m + 16 = 0$

$\therefore m = -4$ and $m = -4$

$\therefore \boxed{y = c_1 e^{-4x} + c_2 x e^{-4x}}$

12. $2y'' + 2y' + y = 0$

The auxiliary equation is

$2m^2 + 2m + 1 = 0$

$\therefore m = \frac{-1}{2} \pm \frac{i}{2}$

$\boxed{\therefore y = e^{-\frac{x}{2}} \left[c_1 \cos\frac{x}{2} + c_2 \sin\frac{x}{2} \right]}$

Exercises 3.4 – UNDETERMINED COEFFICIENTS

3. $y'' - 10y' + 25y = 30x + 3$

The auxiliary equation is $m^2 - 10m + 25 = 0$

$\therefore m_1 = m_2 = 5$

$\therefore y_c = C_1 e^{5x} + C_2 x e^{5x}$

Assume $y_p = Ax + B$

$\therefore y'_p = A$

$y''_p = 0$

Substituting in the equation,

$0 - 10A + 25(Ax + B) = 30x + 3$

$\therefore 25A = 30$ & $-10A + 25B = 3$

$\therefore A = \frac{6}{5}$ and $B = \frac{3}{5}$

$\therefore y_p = \frac{6}{5}x + \frac{3}{5}$

$\therefore \boxed{y = C_1 e^{5x} + C_2 e^{5x} \cdot x + \frac{6}{5}x + \frac{3}{5}}$

10. $y'' + 2y' = 2x + 5 - e^{-2x}$

The auxiliary equation is $m^2 + 2m = 0$

$\therefore m_1 = -2$, and $m_2 = 0$

$\therefore y_c = C_1 e^{-2x} + C_2$

Assume $y_p = Ax^2 + Bx + Cxe^{-2x}$

$\therefore y'_p = 2Ax + B + Ce^{-2x} + (-2)xCe^{-2x}$

$y''_p = 2A + (-2)Ce^{-2x} - 2Ce^{-2x} + 4xCe^{-2x}$

Substituting in the equation,

$2A - 2Ce^{-2x} - 2Ce^{-2x} + 4xCe^{-2x} + 4Ax + 2B + 2Ce^{-2x}$
$-4Cxe^{-2x} = 2x + 5 - e^{-2x}$

$\therefore (2A+2B) + 4Ax - 2Ce^{-2x} = 5 + 2x - e^{-2x}$

$\therefore 4A = 2 \quad \therefore A = \frac{1}{2}$

$2A+2B = 5 \quad \therefore B = 2$

$\& \ -2C = -1 \quad \therefore C = \frac{1}{2}$

$\therefore y_p = \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}$

$$\therefore \boxed{y = C_1e^{-2x} + C_2 + \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}}$$

13. $y'' + 4y = 3\sin 2x$

The auxiliary equation is

$m^2 + 4 = 0$

$\therefore m_1 = 2i$ and $m_2 = -2i$

$\therefore y_c = C_1\cos 2x + C_2\sin 2x$

Assume, $y_p = Ax\cos 2x + Bx\sin 2x$

$\therefore y'_p = A\cos 2x - 2Ax\sin 2x + B\sin 2x + 2Bx\cos 2x$

$\& \ y''_p = -2A\sin 2x - 2A\sin 2x - 4A\cos 2x + 2B\cos 2x$
$+ 2B\cos 2x - 4Bx\sin 2x$

Substituting in the equation,

$-4A\sin 2x - 4Ax\cos 2x + 4B\cos 2x - 4Bx\sin 2x$
$+ 4Ax\cos 2x + 4Bx\sin 2x = 3\sin 2x$

$\therefore -4A\sin 2x + 4B\cos 2x = 3\sin 2x$

$\therefore -4A = 3 \therefore A = -\frac{3}{4}$ and $4B = 0 \therefore B = 0$

$\therefore y_p = -\frac{3}{4}x\cos 2x$

$\therefore \boxed{y = c_1\cos 2x + c_2\sin 2x - \frac{3}{4}x\cos 2x}$

Exercises 3.6 – Cauchy Euler Equation

10. $4x^2y'' + 4xy' - y = 0$
Consider $y = x^m$ $\therefore y' = mx^{m-1}$ & $y'' = m(m-1)x^{m-2}$
Substituting in the equation,
$4x^2[m(m-1)x^{m-2}] + 4x(mx^{m-1}) - \overset{x^m}{y} = 0$

$\therefore 4m(m-1) + 4m - 1 = 0$
$\therefore 4m^2 - 1 = 0$
$\therefore m_1 = \frac{1}{2}$ and $m_2 = -\frac{1}{2}$

$\therefore \boxed{y = c_1x^{1/2} + c_2x^{-1/2}}$

11. $x^2y'' + 5xy' + 4y = 0$

Consider $y = x^m$ $y' = mx^{m-1}$ & $y'' = m(m-1)x^{m-2}$

$x^2[m(m-1)x^{m-2}] + 5xmx^{m-1} + 4x^m = 0$

$\therefore m^2 + 4m + 4 = (m+2)^2 = 0$

$\therefore m_1 = -2$ and $m_2 = -2$

$\therefore y = c_1x^{-2} + c_2x^{-2}\ln x$