MAE 3360

HOMEWORK #5

KEY ASSIGNMENT

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MAE 3360
HW # 5 (Key Assignment)

4. $m = \frac{24}{32} = \frac{3}{4}$ slug

$K = 72$

$\therefore \frac{3}{4}x'' + 72x = 0$

$\therefore \omega = \sqrt{96} = 4\sqrt{6}$

$\therefore x(t) = C_1 \cos(4\sqrt{6}t) + C_2 \sin(4\sqrt{6}t)$

$x(0) = 0 = C_1 \quad \therefore C_1 = 0$

$x'(t) = -4\sqrt{6}\, C_1 \sin(4\sqrt{6}t) + 4\sqrt{6}\, C_2 \cos(4\sqrt{6}t)$

$x'(0) = 2$

$\therefore C_2 = \frac{2}{4\sqrt{6}} = \frac{1}{2\sqrt{6}}$

$$\therefore \boxed{x(t) = \frac{\sin(4\sqrt{6}t)}{2\sqrt{6}}}$$

9. $m = \frac{8}{32} = \frac{1}{4}$ slug $K = 1$

$\frac{1}{4}x'' + x = 0$

$x'' + 4x = 0$

$\therefore \omega = \sqrt{4} = 2$

$\therefore x(t) = C_1 \cos 2t + C_2 \sin 2t$

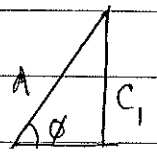
$x(0) = \frac{1}{2}$ & $x'(0) = \frac{3}{2}$

$x(0) = C_1 \cos 2(0) = \frac{1}{2} \quad \therefore C_1 = \frac{1}{2}$

$x'(t) = -2C_1 \sin 2t + 2C_1 \cos 2t$

$x'(0) = \frac{3}{2} \quad \therefore C_2 = \frac{3}{4}$

$x(t) = \frac{1}{2}\cos 2t + \frac{3}{4}\sin 2t$



$A = \sqrt{C_1^2 + C_2^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2} = \frac{\sqrt{13}}{4}$

$\phi = \tan^{-1}\left(\frac{C_1}{C_2}\right) = \tan^{-1}\left(\frac{1/2}{3/4}\right) = 0.588$

$\therefore x(t) = A \sin(\omega t + \phi)$

$\therefore x(t) = \frac{\sqrt{13}}{4}\sin(2t + 0.588) \quad (\omega = 2)$

or $x(t) = \frac{\sqrt{13}}{4}\sin\left(2t + \tan^{-1}\left(\frac{1/2}{3/4}\right)\right)$

21. $m = \frac{4}{32} = \frac{1}{8}$ slug

$\frac{1}{8}x'' + x' + 2x = 0$

$x'' + 8x' + 16x = 0$

$\therefore m^2 + 8m + 16 = 0$

$x(t) = c_1 e^{-4t} + c_2 t e^{-4t}$

$x(0) = -1 = c_1$

$x'(t) = -4c_1 e^{-4t} + c_1 e^{-4t} - 4c_2 t e^{-4t}$

$x'(0) = 8$

$\therefore x(t) = e^{-4t}(-1+4t)$

$x(t) = 0$

$0 = e^{-4t}(-1+4t)$

$\therefore \boxed{t = \frac{1}{4} \text{ sec}}$

$x'(t) = 0$

$0 = 4e^{-4t} + 4e^{-4t} - 16te^{-4t}$

$8e^{-4t} = 16te^{-4t}$

$\boxed{t = \frac{1}{2} \text{ sec}}$

$x\left(\frac{1}{2}\right) = e^{-4\left(\frac{1}{2}\right)}\left(-1+4\left(\frac{1}{2}\right)\right)$

$= e^{-2}(-1+2) = e^{-2}$

$\therefore \boxed{x = e^{-2} \text{ feet}}$

23. $m = 1\text{kg}$ $k = 16\ \text{N/m}$

$x'' + 10x' + 16x = 0$

$m^2 + 10m + 16 = 0$

$m_1 = -8,\ m_2 = -2$

$2\lambda = 10 \Rightarrow \lambda = 5 \quad \therefore \lambda^2 - \omega^2 = 9$ (overdamped)

$x(t) = e^{-5t}(C_1 e^{3t} + C_2 e^{-3t})$

$x'(t) = -5e^{-5t}(C_1 e^{3t} + C_2 e^{-3t}) + 3e^{-5t}(C_1 e^{3t} - C_2 e^{-3t})$

$x(0) = 1$

$x'(0) = 0$

$\therefore -5(C_1 + C_2) + 3(C_1 - C_2) = 0$

~~$\therefore$ [illegible] $= -5$ [illegible]~~

~~$0 = -5(C_1 + C_2$~~ $\quad \therefore C_1 = -4C_2$

$C_1 + C_2 = 1$

$\therefore C_2 = \frac{-1}{3} \quad C_1 = \frac{4}{3}$

$x(t) = e^{-5t}\left(\frac{4}{3}e^{3t} - \frac{1}{3}e^{-3t}\right) = \frac{4}{3}e^{-2t} - \frac{1}{3}e^{-8t}$

b.) $x(0) = 1$

$x'(0) = -12$

$C_1 + C_2 = 1$

$-2C_1 - 8C_2 = -12$

Solving, $C_2 = \frac{5}{3}$

and $C_1 = \frac{-2}{3}$

$\therefore x(t) = -\frac{2}{3}e^{-2t} + \frac{5}{3}e^{-8t}$

25. $m = \frac{3.2}{32} = 0.1$ slug

$k = 2$; $\omega^2 = \frac{k}{m} = 20$

$\therefore \omega = 4\sqrt{5}$ $\lambda^2 - \omega^2 = -16 < 0$ (underdamped)

$0.1x'' + 0.4x' + 2x = 0$

$x'' + 4x' + 20x = 0$

$x(0) = -1$

$x'(0) = 0$

$x(t) = e^{-2t}(C_1 \cos 4t + C_2 \sin 4t)$

$x'(t) = -2e^{-2t}(C_1 \cos 4t + C_2 \sin 4t) + 4e^{-2t}($
$C_2 \cos 4t - C_1 \sin 4t)$

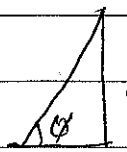
$x(0) = -1 = C_1$

$x'(0) = -2(-1) + 4C_2 = 0$

$\therefore C_2 = -\frac{1}{2}$

$x(t) = e^{-2t}(-\cos 4t - \frac{1}{2}\sin 4t)$

b.) $A = \sqrt{C_1^2 + C_2^2}$ $\tan\phi = \frac{C_1}{C_2}$ $\therefore \phi = 1.107$



$\therefore A = \sqrt{(-1)^2 + (-1/2)^2} = \frac{\sqrt{5}}{2}$

$x(t) = \frac{\sqrt{5}}{2} e^{-2t} \sin(4t + 4.248)$

c.) $x(t) = \frac{\sqrt{5}}{2} e^{-2t} \sin(4t + 4.248) = 0$

$\therefore \boxed{t = 1.294 \text{ s}}$