MAE 3360

HOMEWORK #8

EXERCISES 9.5 - Directional Derivative

12. $f(x,y) = 4x + xy^2 - 5y \quad ; (3,-1), \theta = \frac{\pi}{4}$

$Df(x,y) = (4+y^2)i + (2xy-5)j$

$Df(3,-1) = 5i + -11j$

at $\theta = \frac{\pi}{4}$

$u = \frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$

$Duf(3,-1) = \frac{5\sqrt{2}}{2} - \frac{11\sqrt{2}}{2} = -3\sqrt{2}$

14. $f(x,y) = \frac{xy}{(x+y)} \quad ; (2,-1)$

$Df(x,y) = \frac{y^2}{(x+y)^2}i + \frac{x^2}{(x+y)^2}j$

$Df(2,-1) = i + 4j$

$u = \frac{6}{10}i + \frac{8}{10}j = \frac{3}{5}i + \frac{4}{5}j$

$D_u f(4,-1) = \frac{3}{5} + \frac{16}{5} = \frac{19}{5}$

18. $f(x,y,z) = \frac{x^2 - y^2}{z}$; $(2,4,-1)$, $i - 2j + k$

$\nabla f = \frac{2x}{z^2}i - \frac{2y}{z^2}j - \frac{(2x^2 - 2y^2)}{z^3}k$

$\nabla f(2,4,-1) = 4i - 8j + (-24)k$

$u = \frac{i}{\sqrt{6}} - \frac{2}{\sqrt{6}}j + \frac{1}{\sqrt{6}}k$

$D_u f = \frac{4}{\sqrt{6}} + \frac{16}{\sqrt{6}} - \frac{24}{\sqrt{6}} = \frac{-4}{\sqrt{6}}$

20. $f(x,y,z) = 2x - y^2 + z^2$; $(4,-4,2)$, in the direction of the origin

$\nabla f = 2i - 2yj + 2zk$

$\nabla f(4,-4,2) = 2i + 8j + 4k$

$$u = \frac{(0-4)i + (0-(-4))k + (0-2)k}{\sqrt{4^2 + (-4)^2 + 2^2}}$$

$$\therefore u = \frac{-4i}{6} + \frac{4j}{6} - \frac{2k}{6}$$

$$\therefore Duf = \frac{-4}{3} + \frac{16}{3} - \frac{4}{3} = \frac{8}{3}$$

24. $f(x,y) = xye^{x-y}$; $(5,+5)$

$$\nabla f = (xye^{x-y} + ye^{x-y})i + (-xye^{x-y} + xe^{x-y})j$$

$$\nabla f_{(5,5)} = 30i - 20j$$

For max ~~Du~~ rate $\sqrt{30^2 + (-20)^2} = 10\sqrt{13} = 36.05$
in the direction $30i - 20j$

26. $f(x,y,z) = xyz$; $(3,1,-5)$

$$\nabla f = yzi + xzj + xyk$$

$$\nabla f_{(3,1,-5)} = -5i - 15j + 3k$$

For maximum rate $\sqrt{(-5)^2 + (-15)^2 + 3^2} = 16.09$
in the direction $-5i - 15j + 3k$

31. $f(x,y) = x + y^2$ at $(3,4)$ in direction of a tgt vector to the graph of $2x^2 + y^2 = 9$ at $(2,1)$ (4)

Eqn of tgt : $2x^2 + y^2 = 9 \Rightarrow y^2 = 9 - 2x^2$

Slope of tgt : $\frac{dy}{dx} \Rightarrow 2y\frac{dy}{dx} = -4x$

$$\frac{dy}{dx} = -\frac{2x}{y}$$

Slope of tgt at $(2,1) = -4$

$\therefore u = \pm (i - 4j)/\sqrt{17}$

$\nabla f = i + 2yj$

$\nabla f_{(3,4)} = i + 8j$

$\therefore D_u = \pm \left(\frac{1}{\sqrt{17}} - \frac{32}{\sqrt{17}} \right) = \pm \frac{31}{\sqrt{17}}$

39. $T = 5 + 2x^2 + y^2$

$\nabla T = 4xi + 2yj$

$\nabla T_{(4,2)} = 16i + 4j$

To cool off as rapidly as possible :—

Max decrease in the temp. is in the direction $-\nabla T = -16i - 4j$

EXERCISES 9.6 – Tangent Planes & Normal Lines

16. $F(x,y,z) = 5x^2 - y^2 + 4z^2$

$\nabla F = 10xi - 2yj + 8zk$

$\nabla F_{(2,4,1)} = 20i - 8j + 8k$

∴ Eqn of tgt plane is : $20(x-2) - 8(y-4) + 8(z-1) = 0$

∴ $5x - 2y + 2z = 4$

18. $F(x,y,z) = xy + yz + zx$

$\nabla F = (y+z)i + (z+x)j + (x+y)k$

$\nabla F_{(1,-3,-5)} = -8i - 4j - 2k$

∴ Eqn of tgt plane is: $-8(x-1) - 4(y+3) - 2(z+5) = 0$

$-8x + 8 - 4y - 12 - 2z - 10 = 0$

$\therefore 4x + 2y + z = -7$